RCDue – Laboratory System for Teaching Automation and Control – concept of the system

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Abstract—**Paper describes an implementation of a low cost but nontrivial laboratory system for automation and control theory teaching purposes. The system called RCDue consists of two parts – a dynamic system module and a control unit providing measurement, control and communication with MATLAB through USB serial port. Wiring of the dynamic system module can be modified to get a system of the desired behavior. Application example named as R5C4 with four RC circuits is described together with its mathematical model and calculated step responses. The system can be used in laboratory for practical education of C or MATLAB programming, modelling and identification or control theory subjects.**

Keywords—automation, control, education, teaching, laboratory, experiment, MATLAB

I. Introduction

It is always very beneficial for any teaching purpose if students can verify the acquired theoretical knowledge on a practical example. This is a natural part of education at technical schools, where laboratories are an integral part of the teaching process. In the field of Automation and Control Theory, there is a problem with availability of devices that could be used to easily demonstrate the procedures and methods described in the theory. Using real equipment as a technological unit or a machine in the laboratory is possible but very demanding. Therefore, downscaled or imaginary laboratory models are used.

There are plenty of different companies dealing with educational models e.g. National Instruments Corp. [1], TecQuipment Ltd. [2], G.U.N.T. Gerätebau GmbH [3], Festo Didactic Ltd. [4]. E.g. models from company Humusoft (in cooperation with TecQuipment) [5] are designed for control purposes and can be operated directly from the MATLAB/Simulink environment. Use of professional laboratory models is relatively expensive, and it is assumed that either additional standard industrial instrumentation or a proprietary interface supplied with the model is used. A program containing a set of several prepared tasks is usually part of such a closed solution. Although a connection using standard instrumentation is open and advantageous for gaining practical experience, it increases the cost of the equipment and requires extensive knowledge of the used hardware. The behavior of the model from the control point of view is usually either very simple (first-order dynamics) or on the other hand quite complex and frequently combined with nonlinear behavior.

A lot of academics are designing and fabricating their own laboratory systems based on different hardware and software solutions. Let us name the portable laboratory device Flexy2 [6] developed at the Institute of Information Engineering, Automation and Mathematics, STU in Bratislava which is very similar to our solution. It is also built on Arduino UNO microcontroller and allows "microcontroller embedded" or "MATLAB controlled" operation.

This article aims to describe an easy-to-implement and inexpensive (approximately 50 €) laboratory model referred to as RCDue allowing direct connection to a computer and easy access from the MATLAB environment. The model, including the interface, is small, with minimal power consumption, easy to operate and safe, suitable for use in common classroom so every student can experiment individually.

The difference to conventional laboratory models is the openness of the solution, which allows modifications for different educational tasks. Flexibility is given by the possibility to change the properties of the electrical system (controlled system) and the program in the control unit. Simple connection with the MATLAB environment allows the use of sophisticated methods for data analysis and very complex control algorithms. Example of the simplest task can be static characteristic measurement or step response measurement, where students create script with use of the pre-prepared functions and carry out measurement and data evaluation. On the other hand, advanced students can practice real-time application, communication and sophisticated control algorithms.

The paper is conceived as an introductory part of a series of papers dealing with individual students' tasks. It describes the device and presents an example of creating of the first principle model of one system module. The mathematical model is necessary for every advanced control method. Calculated and measured transient and frequency characteristics of the system are compared to verify the model.

The paper is structured as follows: after the introduction in chapter I., areas of possible applications are named in chapter II, R5C4 version of the laboratory system is described in chapter III. – dynamic system module, control unit and software concept. Mathematical model of the R5C4 module is derived in chapter IV. Model is verified in chapter V and conclusions are given in chapter VI.

II. Areas of Possible Applications

The model in combination with MATLAB can be used to demonstrate different tasks e.g. in the following areas:

1. tasks of step response and frequency characteristics measuring, ultimate gain and period evaluation, determination of transfer function approximation

2. task of implementation and setting of PSD controller, anti-windup methods

3. demonstration of the ambiguity of a state model, two different state models both with states that have physical significance

4. task of experimental identification of parameters of a continuous mathematical model

5. task of off-line and on-line experimental identification of the parameters of the difference equation

6. pole assignment controller design and its implementation

7. state-estimation of a discrete-time state-space model - comparing the estimated state with a measured state (or at least some state variables)

8. LQG controller design and its implementation

9. predictive controller design with constrained variables and its implementation.

III. RCDue TECHNICAL DESCRIPTION

The RCDue laboratory model consists of two separate parts - a dynamic system module and a control unit providing measurement, control if necessary and communication. The connection of the control unit to the computer is through a standard USB cable, which also provides power to both the control unit and the dynamic system. Serial communication uses a simple character-oriented protocol that is easy to implement both in the control unit and in MATLAB.

A. Dynamic system module

As an example, a module of a dynamic system suitable for measuring the frequency characteristics (referred to as R5C4) is described. The system consists of four RC circuits with a load resistor at the output - see Fig. 1.
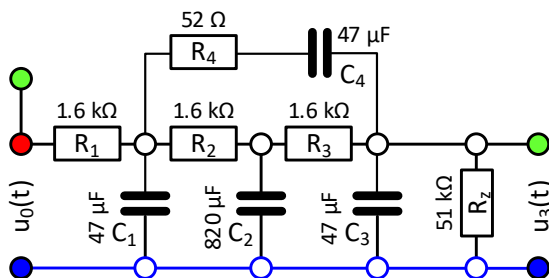

Fig. 1. R5C4 - wiring diagram

Using of passive elements does not require separate power supply, are cheap and do not require additional equipment (breadboard can be easily used). Such wiring also allows voltage measurement at any connection point. If we consider ideal components, then it represents a 4th order linear dynamic system. The R5C4 module was implemented using a breadboard. The scheme of components' placement and connection of the control unit signals is shown in Fig. 2 and its physical appearance is shown in Fig. 3.

Slightly non-standard circuit shows a non-trivial shape of the transient characteristics and a rather atypical shape of the frequency characteristics (positive phase shift) - see Fig. 4. When using components of the given parameters, the atypical region of the phase characteristics is in the region around 1 Hz.
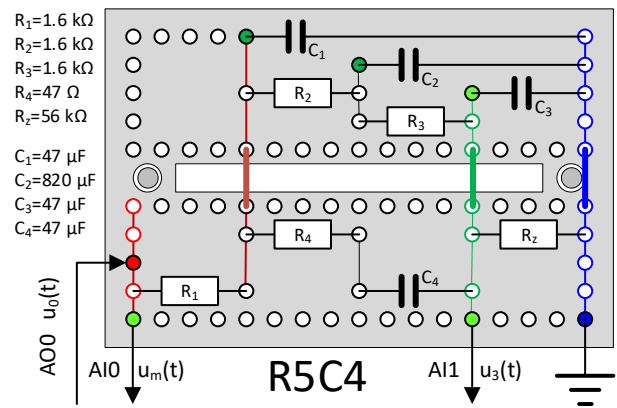

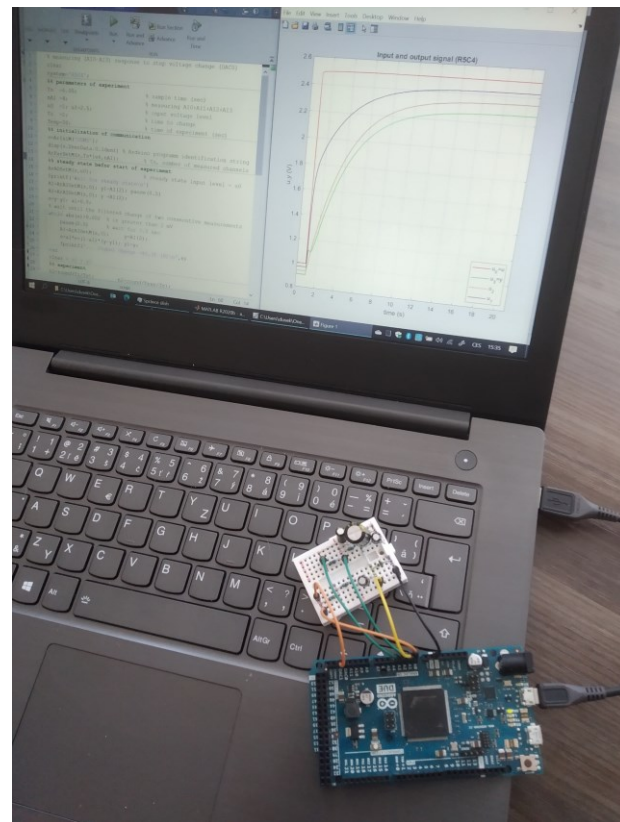Fig. 2. R5C4 – dynamic module breadboard wiring

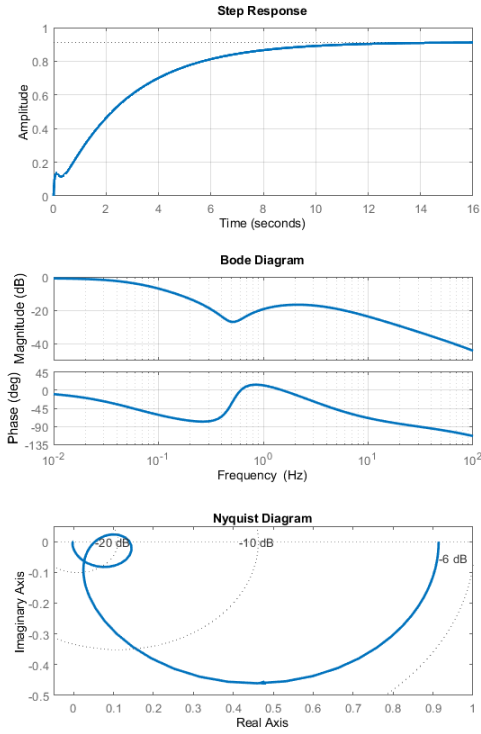Fig. 3. R5C4 – dynamic module, control unit, SW in MATLAB

Fig. 4. R5C4 – time and frequency characteristics

B. Control unit

The measurement of frequency characteristics has the greatest demands on HW equipment. In order to be able to measure the time shift in milliseconds with sufficient accuracy without the need for additional instrumentation, an Arduino Due development board is used. It is equipped with a fast 32-bit RISC processor of the ARM Cortex M3 architecture at 80 MHz, 96 MB of SRAM memory and 512 MB of program flash memory. It is important that in addition to a 12-bit ADC (12 channels), the unit also contains two 12-bit DACs. A large capacity of RAM is necessary to store a sufficient number of samples of the measured signals - at least two periods are recommended to evaluate the time shift. At a high sampling frequency, the measured samples cannot be continuously transferred to a computer and must be stored in memory.

In addition to these HW features, the Arduino Due development board is readily available, inexpensive, and supported by the free Arduino IDE [5] with a number of expanding libraries and a large user community. The Due-Timer library [6] was used in the creation of the program. It enables the use of all nine 32-bit processor timers for periodic execution of program parts with an accuracy of 1 μs. For HW realization of the complete laboratory system it is only necessary to place the resistors and capacitors in the breadboard, connect one DAC to the system input, connect two ADC channels (one to measure real input voltage, the other to measure voltage on the load resistor) and connect Arduino with USB cable to a computer. Everything else is a matter of the program running in Arduino and a script in MATLAB.

C. RCDue - software concept

Due to the simplicity of uploading the program from the development environment to the control unit, it is reasonable to create different programs for different uses of the laboratory

model. The design of the program and communication is different in the case of high measurement speed, when it is necessary to store data in memory and transfer a large amount of data at once or in the case that the control unit responds only to requests to measure and/or set signal values. The implementation of computational algorithms in the control unit also requires a separate program. It is also possible to use MATLAB's ability to generate source code in "C" of user functions and most standard functions. This code can be used directly with minor modifications in the program for control unit [7].

The communication protocol used is the same for all programs. A character-oriented protocol with a terminating pair of CR/LF characters is used to easily solve the communication of Arduino and MATLAB. One character (letter) at the beginning of the message is sufficient due to the limited number of different messages. A colon is inserted after the first character to check that this is really the beginning of the message. All characters between the colon and the CR/LF characters form the message text. The interpretation of the text depends on the initial character. Because the body of a message usually consists of multiple numbers (some with a dot as a decimal separator), the convention is that each number is terminated by a comma. This convention allows easy creation and processing of a string containing multiple numbers.

IV. RCDUE MATHEMATICAL MODEL

The following section is an example of creating a first principle model using matrix notation for the R5C4 module.

Assuming an ideal behavior of the components the system of RC circuits allows a relatively simple creation of a linear dynamic model by the application of Kirhoff's laws, i.e. by the method of mathematical and physical analysis. The matrix form is the expression of a state-space model with states that have physical meaning. In above mentioned connection (module R5C4) there are three voltages out of four directly measurable by the control unit.

Let us denote the voltages on the capacitors as u_1 - u_4, the currents flowing through the capacitors as i_1 - i_4 and the fictitious currents in all loops as i_A - i_D - see Fig. 6.
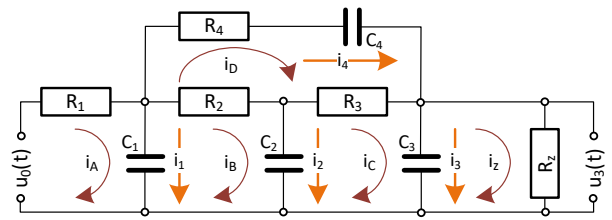

Fig. 5. R5C4 - wiring diagram with variables

We can use the loop current method (the sum of currents in each loop is zero) together with the relationship for the voltage drop on the capacitor and the resistor

$$u_C = \frac{1}{C}\int_0^t i\,dt,\ u_R = Ri \tag{1}$$

to formulate a basic description in the form of a system of 5 following equations

$$u_0 = R_1 i_A + \frac{1}{C_1}\int_0^t (i_A - i_B)dt,$$

$$i_1 = i_A - i_B, \quad i_A = i_1 + i_2 + i_3 + i_z \tag{2}$$

$$0 = R_2(i_B - i_D) + \frac{1}{C_2}\int_0^t (i_B - i_C)dt + \frac{1}{C_1}\int_0^t (i_B - i_A)dt,$$

$$i_2 = i_B - i_C, \quad i_B = i_2 + i_3 + i_z \tag{3}$$

$$0 = R_3(i_C - i_D) + \frac{1}{C_3}\int_0^t (i_C - i_z)dt + \frac{1}{C_2}\int_0^t (i_C - i_B)dt,$$

$$i_3 = i_C - i_z, \quad i_C = i_3 + i_z \tag{4}$$

$$0 = R_4 i_D + \frac{1}{C_4}\int_0^t i_D dt + R_2(i_D - i_B) + R_3(i_D - i_C),$$

$$i_4 = i_D, \quad i_D = i_4 \tag{5}$$

$$0 = R_z i_z + \frac{1}{C_3}\int_0^t (i_z - i_C)dt,$$

$$i_z = \frac{1}{R_z}u_3 \tag{6}$$

After excluding the loop currents i_{A-D}, these equations can be converted into a matrix expression (7) describing the time dependence of the currents i_{1-4}, i_z and the voltages u_{1-4} on the course of the voltage u_0. The matrix expression significantly simplifies and clarifies used modifications. The resulting matrix expressions directly applicable in MATLAB are

$$\mathbf{Ii} = \mathbf{Uu} + \mathbf{m}u_0, \quad \mathbf{K}\frac{d\mathbf{u}}{dt} = \mathbf{i} \tag{7}$$

where

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}, \quad \mathbf{m} = \begin{bmatrix} \frac{1}{R_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & R_2 & R_2 + R_3 & -R_2 - R_3 - R_4 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} -\frac{1}{R_1} & 0 & -\frac{1}{R_z} & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2} & -\frac{1}{R_z} & 0 \\ 0 & \frac{1}{R_3} & -\frac{1}{R_3} - \frac{1}{R_z} & 0 \\ 0 & 0 & -\frac{R_2 + R_3}{R_z} & 1 \end{bmatrix}$$

We can express either currents or voltages from equation (7). By excluding the currents from (7) we obtain a "voltage" state-space model (9) describing the dependence of the output voltage u_3 on the input voltage u_0 by means of the voltage vector $\mathbf{u}$ on the capacitors as a state vector.

$$\mathbf{i} = \mathbf{I}^{-1}\mathbf{Uu} + \mathbf{I}^{-1}\mathbf{m}u_0, \quad \frac{d\mathbf{u}}{dt} = \mathbf{K}^{-1}\mathbf{i} \tag{8}$$

$$\frac{d\mathbf{u}}{dt} = \underbrace{\mathbf{K}^{-1}\mathbf{I}^{-1}\mathbf{U}}_{\mathbf{A}_u}\mathbf{u} + \underbrace{\mathbf{K}^{-1}\mathbf{I}^{-1}\mathbf{m}}_{\mathbf{b}_u}u_0$$

$$u_3 = \underbrace{[0 \quad 0 \quad 1 \quad 0]}_{\mathbf{c}_u}\mathbf{u} \tag{9}$$

where

$$\mathbf{A}_u = \begin{bmatrix} -\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) & \frac{1}{C_1 R_2} & \frac{1}{C_1 R_4} & \frac{1}{C_1 R_4} \\ \frac{1}{C_2 R_2} & -\frac{1}{C_2}\left(\frac{1}{R_2} + \frac{1}{R_3}\right) & \frac{1}{C_2 R_3} & 0 \\ \frac{1}{C_3 R_4} & \frac{1}{C_3 R_3} & -\frac{1}{C_3}\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_z}\right) & -\frac{1}{C_3 R_4} \\ \frac{1}{C_4 R_4} & 0 & -\frac{1}{C_4 R_4} & -\frac{1}{C_4 R_4} \end{bmatrix}$$

$$\mathbf{b}_u = \begin{bmatrix} \frac{1}{C_1 R_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{c}_u = [0 \quad 0 \quad 1 \quad 0]$$

By introducing modified currents $\mathbf{i}^*$ into the matrix form to exclude the voltages we obtain

$$\mathbf{i}^* = \mathbf{Ii} - \mathbf{m}u_0, \quad \mathbf{i} = \mathbf{I}^{-1}(\mathbf{i}^* + \mathbf{m}u_0) \tag{10}$$

Voltage $\mathbf{u}$ from (7) gives

$$\mathbf{u} = \mathbf{U}^{-1}(\mathbf{Ii} - \mathbf{m}u_0) = \mathbf{U}^{-1}\mathbf{i}^* \tag{11}$$

and its derivation is

$$\frac{d\mathbf{u}}{dt} = \mathbf{U}^{-1}\frac{d\mathbf{i}^*}{dt} = \mathbf{K}^{-1}\mathbf{i} = \mathbf{K}^{-1}\mathbf{I}^{-1}(\mathbf{i}^* + \mathbf{m}u_0) \tag{12}$$

Derivation of modified currents $\frac{d\mathbf{i}^*}{dt}$ represents the "current" state-space model describing the dependence of the output voltage u_3 on the input voltage u_0 by means of the modified currents $\mathbf{i}^*$ as a state vector

$$\frac{d\mathbf{i}^*}{dt} = \underbrace{\mathbf{U}\mathbf{K}^{-1}\mathbf{I}^{-1}}_{\mathbf{A}_i}\mathbf{i}^* + \underbrace{\mathbf{U}\mathbf{K}^{-1}\mathbf{I}^{-1}\mathbf{m}}_{\mathbf{b}_i}u_0$$

$$u_3 = \underbrace{[0 \quad 0 \quad 1 \quad 0]\mathbf{U}^{-1}}_{\mathbf{c}_i}\mathbf{i}^* \tag{13}$$

where

$$\mathbf{A}_i = \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} & A_{i14} \\ A_{i21} & A_{i22} & A_{i23} & A_{i24} \\ A_{i31} & A_{i31} & A_{i31} & A_{i31} \\ A_{i41} & A_{i42} & A_{i43} & A_{i44} \end{bmatrix}$$

$$A_{i11} = \frac{-1}{C_1 R_1}, \quad A_{i12} = \frac{C_3 R_2 R_z - C_1 R_1 R_2 + C_3 R_4 R_z}{C_1 C_3 R_1 R_4 R_z}$$

$$A_{i13} = -\frac{C_1 R_1 R_3 + C_1 R_1 R_4 - C_3 R_3 R_z}{C_1 C_3 R_1 R_4 R_z}, \quad A_{i14} = \frac{C_1 R_1 - C_3 R_z}{C_1 C_3 R_1 R_4 R_z}$$

$$A_{i21} = \frac{1}{C_1 R_2}, \quad A_{i22} = -\frac{C_1 C_2 R_2^2 + C_2 C_3 R_2 R_z + C_1 C_3 R_4 R_z + C_2 C_3 R_4 R_z}{C_1 C_2 C_3 R_2 R_4 R_z}$$

$$A_{i23} = -\frac{C_1 C_2 R_2 R_3 + C_1 C_2 R_2 R_4 - C_1 C_3 R_4 R_z + C_2 C_3 R_3 R_z}{C_1 C_2 C_3 R_2 R_4 R_z}$$

$$A_{i24} = \frac{C_1 R_2 + C_3 R_z}{C_1 C_3 R_2 R_4 R_z}, \quad A_{i44} = \frac{C_4 R_2 + C_4 R_3 - C_3 R_z}{C_3 C_4 R_4 R_z}$$

$$A_{i31} = 0, \quad A_{i32} = -\frac{C_2 R_2 R_3 + C_2 R_2 R_z - C_3 R_4 R_z}{C_2 C_3 R_3 R_4 R_z}$$

$$A_{i13} = -\frac{C_2 R_2^2 + C_2 R_3 R_4 + C_2 R_3 R_z + C_3 R_4 R_z}{C_2 C_3 R_3 R_4 R_z}, \quad A_{i14} = \frac{R_3 + R_z}{C_3 R_3 R_4 R_z}$$

$$A_{i41} = 0, \quad A_{i42} = -\frac{C_4 R_2^2 + C_4 R_2 R_3 - C_3 R_2 R_z}{C_3 C_4 R_4 R_z}$$

$$A_{i43} = -\frac{C_4 R_3^2 + C_4 R_2 R_3 + C_4 R_2 R_4 + C_4 R_3 R_4 - C_3 R_3 R_z}{C_3 C_4 R_4 R_z}$$

$$\mathbf{b}_i = \begin{bmatrix} \frac{-1}{C_1 R_1^2} \\ \frac{-1}{C_1 R_1 R_2} \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{c}_i = \begin{bmatrix} \frac{-R_1 R_z}{R_1 + R_2 + R_3 + R_z} & \frac{-R_2 R_z}{R_1 + R_2 + R_3 + R_z} & \frac{-R_3 R_z}{R_1 + R_2 + R_3 + R_z} & 0 \end{bmatrix}$$

Calculated response of the "voltage" model (9) to a step on the input voltage u_0 is plotted in Fig. 6. Response of the

"current" model (13) to is plotted in Fig. 7. Modified currents are recalculated to currents **i** according to (10).
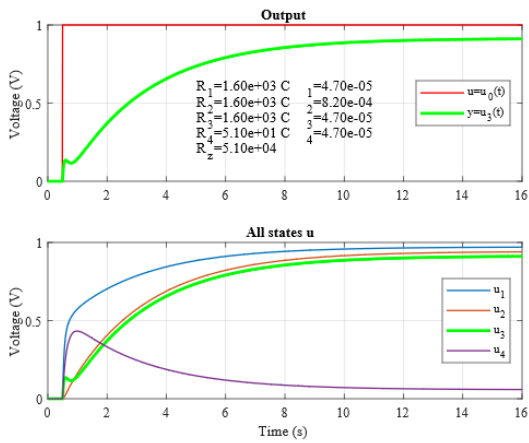

Fig. 6. R5C4 - calculated step responses – model with voltages
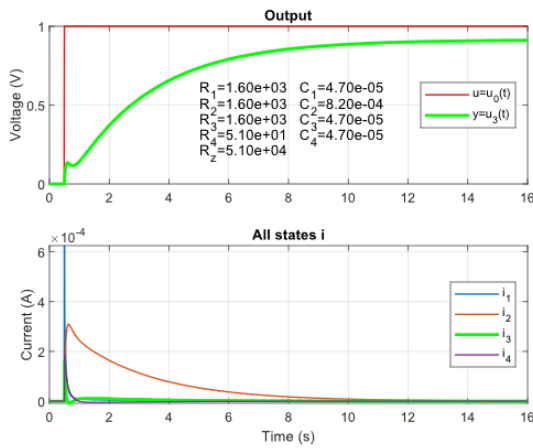

Fig. 7. R5C4 - calculated step responses – model with currents

V. MODEL VERIFICATION

R5C4 dynamic model version is verified by measured step and frequency responses – see Fig. 8 and Fig. 9. The voltages in Fig. 9 - an example of one point measurement of Bode diagram at frequency 0.778 Hz are plotted without offset. In can be seen that the model describes real system well and it can be used for simulations and simple controller design methods. For use of advanced model-based controllers it is necessary to identify the parameters of the model to describe real system behavior better.
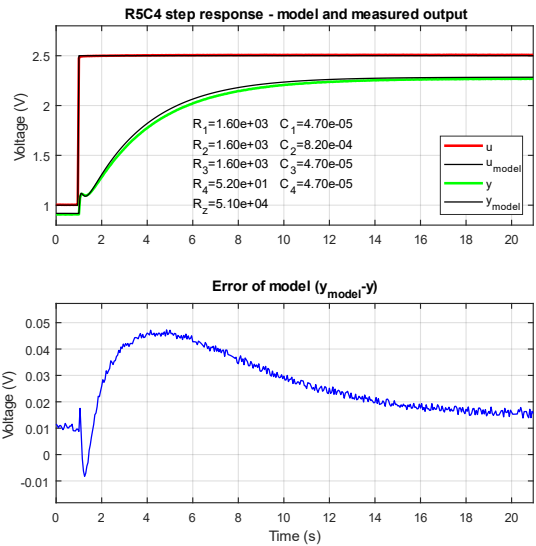

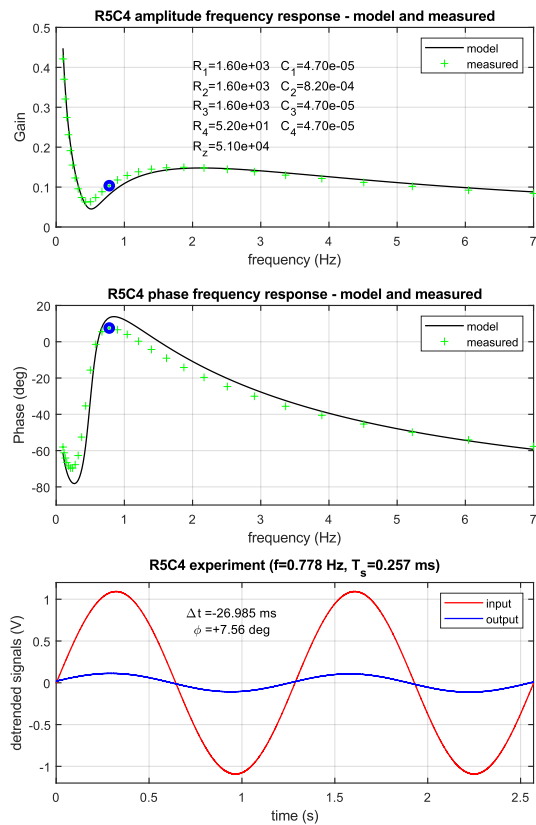Fig. 8. R5C4 – verification through step response

Fig. 9. R5C4 – verification through frequency responses

VI. CONCLUSION

Authors are presenting inexpensive and configurable laboratory system for automation and control education called RCDue. It consists from two parts – the dynamic part allowing user modifications of wiring to get a desired system behavior and the control unit based on Arduino Due development board used as a controller and/or an interface with PC and MATLAB environment. Authors give example of first principle modelling task of dynamic part called R5C4 together with its

verification through real experiments. Detailed technical description of the control unit and MATLAB software as well as advanced identification and control application examples will follow in future authors' publications.

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