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Picture Fuzzy MCDM Approach for Risk Assessment of Railway Infrastructure

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Abstract: To increase the level of safety and prevent significant accidents, it is essential to prioritize risk factors and assess railway infrastructure. The key question is how to identify unsafe railway infrastructure so authorities can undertake safety improvement projects on time. The paper aims to introduce a picture fuzzy group multi-criteria decision-making approach for risk assessment of railway infrastructure. Firstly, picture fuzzy sets are employed for representing and handling risk-related information. Secondly, a picture fuzzy hybrid method based on the direct rating, and Tsallis–Havrda–Charvát entropy is provided to prioritize risk factors. Thirdly, a picture fuzzy measurement of alternatives and ranking according to compromise solution method is developed to rank railway infrastructures. Lastly, the formulated approach is implemented in the Czech Republic context. Two sensitivity analyses verified the high robustness of the formulated approach. The comparative analysis with five state-of-the-art picture fuzzy approaches approved its high reliability. Compared to the state-of-the-art picture fuzzy approaches, the provided three-parametric approach has superior flexibility.

Keywords: risk; railway infrastructure; multi-criteria decision-making; picture fuzzy set; MARCOS; uncertainty

1. Introduction

Railway infrastructure safety is an important element of urban public safety [1]. The safety of railway infrastructure is vital to secure goods and passengers from departures to destinations [2]. Accidental events in railway transportation cause damages to human health, public property, environment, and the economy [3]. In 2018, there were 1721 significant railway accidents in the European Union, with a total of 853 fatalities and 760 serious injuries. The total cost of railway accidents was approximately 5 billion EUR [4]. Collisions and derailments account for about 200 accidents each year [5].

To increase safety levels of assets, passengers, goods, and employees as well as prevent significant accidents it is essential to prioritize risk factors and assess railway infrastructure. However, the risk assessment of railway infrastructure is an emerging problem for railway planners. It is considered as a multi-criteria decision-making (MCDM) problem, since there are a finite set of available railway infrastructures, which safety levels need to be assessed, and numerous risk factors influencing railway infrastructure (i.e., evaluation criteria). Additionally, to mitigate information loss and prevent erroneous decisions, it is critical to efficiently represent and handle risk-related uncertain information when solving this complex MCDM problem.

Solving the railway infrastructure risk assessment problem is critical for railway transport worldwide. The key question is how to identify unsafe railway infrastructure so relevant authorities can undertake safety improvement projects on time. However, (i) only deterministic numbers and type-1 fuzzy sets have been used in the available studies to evaluate risk factors in railway transport; (ii) there is no methodological framework that can represent and handle uncertain, incomplete, and inconsistent risk-related information; and (iii) the available prioritization methods are unable to consider the subjective and objective importance of risk factors under a highly uncertain environment. As a result, the main objective of this paper is to introduce a picture fuzzy group MCDM approach for risk assessment of railway infrastructure.

Recently, Cuong and Kreinovich [6] and Cuong [7] proposed picture fuzzy sets (PFSs). PFSs represent an advanced generalization of intuitionistic fuzzy sets for efficient uncertainty modeling and solving real-life decision-making problems [8]. They can efficiently describe fuzzy, uncertain, incomplete, and inconsistent information [9]. PFSs are characterized by four functions, namely, the degree of positive membership, the degree of neutral membership, the degree of negative membership, and the degree of refusal membership [10,11]. Therefore, they are especially suitable for decision-making situations that require answers of the type yes, no, abstain, and refusal [12]. This particular type of information is dominantly used in surveys and voting systems where experts are divided into previously mentioned four categories [13]. As a result, PFSs can efficiently represent railway planners' preferences and handle risk-related information. Unfortunately, railway planners are unable to naturally express their preferences by voting, since no previous research has applied a PFS-based MCDM approach for railway transport.

The Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) is one of the most recent MCDM methods developed by Stević et al. [14]. It synthesizes the ratio and reference point sorting approaches to obtain ranking results [15]. Compared with other MCDM methods, the MARCOS method is simple, effective, and easy to sort and optimize a decision-making process [14,16]. Its features are [14,15] consideration of ideal and anti-ideal solutions at the very beginning, the definition of utility degree to both solutions, ability to process large data sets, flexibility to analyze expert preferences, and a simple algorithm. However, the MARCOS method has not been extended before using PFSs, so it cannot reflect neutral/refusal information of decision-makers in the railway industry.

Based on the highlighted research gaps, the aims of this paper are (1) to utilize PFSs, which are superior in handling fuzzy, uncertain, incomplete, and inconsistent risk-related information, and help railway planners to naturally express their risk preferences by voting; (2) to determine the importance of each risk factor influencing railway infrastructure by using the novel picture fuzzy hybrid method for risk factor prioritization, which is developed by hybridizing the picture fuzzy direct rating and Tsallis–Havrda–Charvát entropy methods; (3) to rank railway infrastructures by employing the developed picture fuzzy MARCOS method for railway infrastructure ranking, which is for the first time extended under the picture fuzzy environment; and (4) to apply the formulated picture fuzzy group MCDM approach for risk assessment of railway infrastructure in the Czech Republic.

The rest of the paper is organized as follows: Section 2 surveys the state-of-the-art research. Section 3 reviews some definitions of PFSs. Section 4 presents the introduced picture fuzzy group MCDM approach for risk assessment of railway infrastructure. A real-life case study is described in Section 5. Section 6 presents the case study results and discussions. Section 7 gives the conclusions of the work and indicates possible extension areas.

2. Literature Review

The literature review is organized into three sub-sections. The first sub-section surveys existing MCDM approaches for railway transport. The second sub-section overviews applications and extensions of the MARCOS method. The last sub-section presents identified research gaps.

2.1. MCDM Approach for Railway Transport

The application of the MCDM approach for solving diverse problems in railway transport attracted a large interest of researchers, since it is important for the economy and society. The comprehensive summary of the state-of-the-art contributions is presented in Table 1.

Mohajeri and Amin [17] coupled the Analytical Hierarchy Process (AHP) with Data Envelopment Analysis (DEA) to evaluate sites for a railway station. Sivilevičius and Maskeliūnaite [18] applied the AHP method to assess quality criteria of passenger railway trips in international routes. Brunner et al. [19] utilized the AHP method to compare fixed-rail transit alignment locations plotted in the Geographic Information System (GIS) environment. Liu et al. [20] integrated the analytic network process (ANP) and the technique of multiple regression to elicit the environmental impact of high-speed railway systems. Poorzahedy and Rezaei [21] applied several MCDM methods to rank alternative configurations of a light railway transit network.

Celik et al. [22] developed an interval type-2 fuzzy (VIšeKriterijumska Optimizacija i kompromisno Rešenje) VIKOR method to rank railway transit lines. They hybridized the statistical analysis, service quality (SERVQUAL) model, and direct rating method to classify and acquire criteria weights of passenger satisfaction criteria. Lu et al. [23] provided a simple fuzzy framework to determine critical risk factors in railway reconstruction projects. Nathanail [24] applied the Delphi-AHP approach to evaluate quality indicators of a railway network.

Aydin et al. [25] coupled the statistical analysis, fuzzy AHP method, and Choquet integral to compare railway transit lines from the passenger satisfaction perspective. Dhir et al. [26] used the cost–benefit analysis (CBA) and AHP method to identify the most suitable manufacturer for high-speed rail vehicles. An et al. [27] employed the fuzzy AHP method to obtain contributions of hazard groups in a shunting depot. Zhao et al. [28] used the fuzzy AHP method to prioritize fire emergency alternatives of an unattended train operation metro system.

Hamurcu and Eren [29] utilized the AHP and Technique for the Order Preference by Similarity to Ideal Solution (TOPSIS) methods to select the best monorail technology. Montesinos-Valera et al. [30] exploited the ANP method with benefits, opportunities, costs, and risks (BOCR) analysis to order projects for maintenance, renewal, and improvement of railway lines. Polat et al. [31] applied the fuzzy AHP-TOPSIS approach to compare rail suppliers in intercity construction projects. Song et al. [32] utilized the AHP method and interpretative structural modeling (ISM) approach to ascertain the importance and interrelations of vulnerability factors of urban rail transit operations, respectively. Stević et al. [33] combined the best–worst method (BWM), simple additive weighting (SAW) method, and rough numbers to select used rail wagons for carrying out internal transport.

Hamurcu and Eren [34] used the ANP method to prioritize railway construction projects. Mlinarić et al. [35] employed the Delphi-AHP approach to prioritize a common set of key performance indicators of railway intelligent transportation systems. Sharma et al. [36] combined the rough AHP and Multi-Attributive Border Approximation Area Comparison (MABAC) methods to rank railway stations from the service quality perspective. Wang et al. [37] utilized the AHP method and Fuzzy Reasoning Approach (FRA) to assess safety performances of railway track systems. Yadegari et al. [38] used the Delphi-ANP approach to prioritize factors affecting the development of an industry cluster of the rail industry.

Đorđević et al. [39] formulated a rough FUll Consistency Method (FUCOM) to compute subjective weights of service quality criteria in railway transport from the position of passengers with disabilities. Jasti and Ram [40] used the AHP method to appraise performance indicators and benchmark metro systems. Krmac and Djordjević [41] utilized the AHP method to obtain weights of strengths, weaknesses, opportunities, and threats of implementing rail traffic management systems. Norouzi and Namin [42] applied the fuzzy BWM-TOPSIS approach to sort risks of constructing railway megaprojects. Phanyakit and Satiennam [43] employed the AHP-SAW approach to select a rehabilitation alternative for railway tracks structure. Wu et al. [44] used the fuzzy ANP method to evaluate risks of constructing metro stations.

Recently, Blagojević et al. [45] coupled the DEA and SAW methods to rank railway line sections in terms of safety. Brumercikova and Sperka [46] used the AHP method to obtain an entrance order of freight carriers in railway stations. Huang et al. [47] utilized two objective criteria weighting methods and historical data to elicit risks of transporting dangerous goods by railway. Kumar and Anbanandam [48] applied the grey decision making trial and evaluation laboratory (DEMATEL) method to quantify relationships of prominent inhibitors to the implementation of railroad freight transport. Li et al. [49] presented a hybrid Pythagorean MULTIMOORA method to order rail transit lines according to the passenger satisfaction level. Majumdar et al. [50] exploited the TOPSIS method to prioritize key factors affecting metro rail infrastructure based on commuter perception. Sangiorgio et al. [51] integrated the AHP method and linear programming approach to estimate the number of accidents and assess the safety of railway networks. Stoilova [52] coupled the AHP and sequential interactive model of urban systems (SIMUS) methods to select railway passenger plans. Stoilova et al. [53] used the SIMUS method to compare railway transport systems of different countries. Vesković et al. [54] combined the pivot pairwise relative criteria importance assessment (PIPRECIA) and evaluation based on distance from average solution (EDAS) methods to rank options for achieving business balance in organizing railway traffic. Wang and Shi [55] applied an interval-valued intuitionistic fuzzy weighted average operator and maximum entropy method to prioritize the traffic service quality of urban railway transit lines. Zhang and Sun [56] formulated a Delphi-DEMATEL-ANP-TOPSIS approach to assess shunting derailment risk response strategies in railway stations. Tavassolirizi et al. [57] used the DEMATEL-ANP approach to prioritize key delay factors in implementing railway development projects and identify their interactions.

2.2. Applications and Extensions of the MARCOS Method

The MARCOS method is one of the latest MCDM approaches. Its applications and extensions are summarized in Table 2.

Badi and Pamucar [58] coupled the grey direct rating and MARCOS methods to evaluate suppliers for the iron and steel industry. Ilieva et al. [59] utilized the fuzzy MARCOS method to order cloud storage systems. Chakraborty et al. [60] ranked suppliers for the iron and steel industry based on D numbers and the MARCOS method. Puška et al. [61] used the MARCOS method to rank project management software solutions for facilitating business operations. Stanković et al. [62] developed a fuzzy MARCOS method to identify a road section with the highest traffic risk for all participants. They applied the fuzzy PIPRECIA method to determine the importance of criteria affecting the traffic risk. Stević and Brković [63] combined the FUCOM and MARCOS approach to compare performances of drivers engaged in international road transport. Stević et al. [14] developed the MARCOS method and used it to solve the sustainable supplier selection problem in the healthcare industry. Ulutaş et al. [16] integrated the correlation coefficient and standard deviation (CCSD), indifference threshold-based attribute ratio analysis (ITARA), and MARCOS methods to select manual stacker for performing logistics activities in warehouses.

2.3. Research Gaps

According to the performed comprehensive literature review, the research gaps are as follows: (i) No previous research has applied a PFS-based MCDM approach for railway transport. Hence, railway planners are unable to naturally express their preferences by voting; (ii) there is no risk assessment framework that can handle ambiguous, uncertain, and vague information, since only deterministic numbers and type-1 fuzzy sets have been used in the available studies; (iii) the direct rating and Tsallis–Havrda–Charvát entropy methods have not been hybridized before to determine the importance of risk factors; and (iv) the MARCOS method has not been extended previously using picture fuzzy sets.

Table 1. Summary of the available MCDM approaches for railway transport.

Author(s)	Research Focus	GDM (Yes/No)	Parameter Type	SA (Yes/No)	CA (Yes/No)	Method(s)	Application Type
Mohajeri and Amin [17]	Station location selection	Yes	Deterministic	No	No	AHP, DEA	Real-life
Sivilevičius and Maskeliūnaite [18]	Traffic quality indicator evaluation	Yes	Deterministic	No	No	AHP	Real-life
Brunner et al. [19]	Transit alignment location selection	Yes	Deterministic	No	No	AHP, GIS	Real-life
Liu et al. [20]	System indicator evaluation	No	Deterministic	No	No	ANP, RA, ANOVA	Real-life
Poorzahedy and Rezaei [21]	Network configuration evaluation	Yes	Deterministic	No	Yes	ELECTRE, SAW, TOPSIS	Real-life
Celik et al. [22]	Transit line quality evaluation	Yes	IT2F	Yes	No	SERVQUAL, DR, VIKOR	Real-life
Lu et al. [23]	Reconstruction risk factor evaluation	Yes	Fuzzy	No	No	DR	Real-life
Nathanail [24]	Infrastructure quality indicator evaluation	Yes	Deterministic	No	No	Delphi, AHP	Real-life
Aydin et al. [25]	Transit line quality evaluation	Yes	Fuzzy	No	No	AHP, Choquet integral	Real-life
Dhir et al. [26]	Rolling stock manufacturer evaluation	No	Deterministic	No	Yes	AHP, CBA	Real-life
An et al. [27]	Shunting depot risk factor evaluation	No	Fuzzy	No	No	AHP	Real-life
Zhao et al. [28]	Metro system emergency evaluation	Yes	Fuzzy	No	No	AHP	IE
Hamurcu and Eren [29]	Monorail technology evaluation	No	Deterministic	No	No	AHP, TOPSIS	IE
Montesinos-Valera et al. [30]	Line maintenance project selection	No	Deterministic	Yes	No	ANP, BOCR, DR	Real-life
Polat et al. [31]	Supplier selection	Yes	Fuzzy	No	No	AHP, TOPSIS	IE
Song et al. [32]	System vulnerability factor evaluation	Yes	Deterministic	No	No	AHP, ISM	Real-life
Stević et al. [33]	Wagon evaluation	Yes	Rough	Yes	Yes	BWM, SAW	Real-life
Hamurcu and Eren [34]	Construction project selection	No	Fuzzy	No	No	ANP	Real-life
Mlinarić et al. [35]	ITS performance indicator evaluation	Yes	Deterministic	No	No	Delphi, AHP, SWOT	Real-life
Sharma et al. [36]	Station service evaluation	Yes	Rough	Yes	Yes	AHP, MABAC	Real-life
Wang et al. [37]	Track maintenance option selection	No	Fuzzy	No	No	FRA, AHP	IE
Yadegari et al. [38]	Industry cluster indicator evaluation	Yes	Deterministic	No	No	Delphi, AHP	Real-life
Đorđević et al. [39]	Traffic quality indicator evaluation	Yes	Rough	No	No	FUCOM	Real-life
Jasti and Ram [40]	System performance indicator evaluation	Yes	Deterministic	No	No	AHP	Real-life
Krmac and Djordjević [41]	Train CIS enabler and barrier evaluation	No	Deterministic	Yes	No	SWOT, AHP	IE
Norouzi and Namin [42]	Construction risk factor evaluation	No	Fuzzy	No	No	BWM, TOPSIS	Real-life
Phanyakit and Satiennam [43]	Line maintenance project selection	Yes	Det., fuzzy	No	No	AHP, SAW	Real-life
Wu et al. [44]	Station construction risk factor evaluation	Yes	Fuzzy	No	No	ANP	Real-life
Blagojević et al. [45]	Line section safety evaluation	Yes	Det., fuzzy	Yes	Yes	PIPRECIA, SE, SAW	Real-life
Brumercikova and Sperka [46]	Freight carrier access evaluation	No	Deterministic	No	No	AHP	Real-life
Huang et al. [47]	Transport risk factor evaluation	No	Deterministic	Yes	No	SE, SDM	Real-life
Kumar and Anbanandam [48]	Railroad transport inhibitor evaluation	Yes	Grey	No	No	DEMATEL	Real-life
Li et al. [49]	Transit line quality evaluation	Yes	PyF	No	Yes	DR, SE, MULTIMOORA	Real-life
Majumdar et al. [50]	Metro infrastructure indicator evaluation	Yes	Deterministic	No	Yes	TOPSIS, ISA	Real-life
Sangiorgio et al. [51]	Network safety performance assessment	No	Deterministic	No	No	AHP, LP	Real-life
Stoilova [52]	Traffic plan evaluation	No	Deterministic	Yes	No	AHP, SIMUS	Real-life
Stoilova et al. [53]	Network performance assessment	No	Deterministic	Yes	Yes	SIMUS	Real-life
Vesković et al. [54]	Traffic rationalization option selection	Yes	Fuzzy	Yes	Yes	PIPRECIA, EDAS	Real-life
Wang and Shi [55]	Transit line quality evaluation	Yes	IVIF	Yes	No	ME, IVIFWAO	Real-life
Zhang and Sun [56]	Train derailment response plan evaluation	Yes	Deterministic	No	No	DEMATEL, ANP, TOPSIS	Real-life
Tavassolirizi et al. [57]	Project delay indicator evaluation	Yes	Deterministic	No	No	Delphi, ANP, DEMATEL	Real-life
Our study	*Railway infrastructure risk assessment*	*Yes*	*Picture fuzzy*	*Yes*	*Yes*	*DR, THCE, MARCOS*	*Real-life*

Analysis Of Variance: ANOVA; Analytic Network Process: ANP; Analytical Hierarchy Process: AHP; Benefits, Opportunities, Costs, and Risks: BOCR; Best-Worst Method: BWM; Comparative Analysis: CA; Control Information System: CIS; Cost-Benefit Analysis: CBA; Data Envelopment Analysis: DEA; Decision Making Trial and Evaluation Laboratory: DEMATEL; Direct Rating: DR; Elimination Et Choix Traduisant La Realite: ELECTRE; Evaluation based on Distance from Average Solution: EDAS; Full Consistency Method: FUCOM; Fuzzy Reasoning Approach: FRA; Geographic Information Systems: GIS; Group Decision-Making: GDM; Illustrative Example: IE; Importance Satisfaction Analysis: ISA; Intelligent Transportation System: ITS; Interpretative Structural Modeling: ISM; Interval type-2 fuzzy: IT2F; Interval-Valued Intuitionistic Fuzzy Weighted Average Operator: IVIFWAO; Interval-Valued Intuitionistic Fuzzy: IVIF; Linear Programming: LP; Maximum Entropy: ME; Measurement of Alternatives and Ranking according to Compromise Solution: MARCOS; Multi-Attributive Border Approximation area Comparison: MABAC; Multi-Objective Analysis by Ratio Analysis plus the Full Multiplicative Form: MULTIMOORA; Pivot Pairwise Relative Criteria Importance Assessment: PIPRECIA; Pythagorean fuzzy: PyF; Regression analysis: RA; Scatter Degree Method: SDM; Sensitivity Analysis: SA; Sequential Interactive Model of Urban Systems: SIMUS; Service Quality: SERVQUAL; Shannon Entropy: SE; Simple Additive Weighting: SAW; Strengths, Weaknesses, Opportunities, and Threats: SWOT; Technique for the Order Preference by Similarity to Ideal Solution: TOPSIS; Tsallis–Havrda–Charvát Entropy: THCE; VIšeKriterijumska Optimizacija i kompromisno Rešenje: VIKOR.

Table 2. Summary of the available applications and extensions of the MARCOS method.

Author(s)	Research Focus	GDM (Yes/No)	Parameter Type	SA (Yes/No)	CA (Yes/No)	Criteria Weighting Method(s)			Application Type
						Subjective	Objective	Hybrid (Yes/No)	
Badi and Pamucar [58]	Supplier selection	Yes	Grey, Det.	No	Yes	Direct rating	–	No	Real-life
Chakraborty et al. [60]	Supplier selection	Yes	D number	No	No	Direct rating	–	No	Real-life
Ilieva et al. [59]	Cloud service provider selection	No	Fuzzy	No	No		Not specified		Illustrative example
Puška et al. [61]	Software evaluation	Yes	Deterministic	Yes	No	Direct rating	–	No	Real-life
Stanković et al. [62]	Road traffic risk evaluation	No	Fuzzy	Yes	Yes	PIPRECIA	–	No	Real-life
Stević and Brković [63]	Personnel selection	No	Deterministic	No	Yes	FUCOM	–	No	Illustrative example
Stević et al. [14]	Sustainable supplier selection	Yes	Deterministic	Yes	Yes	Direct rating	–	No	Real-life
Ulutaş et al. [16]	Equipment evaluation	Yes	Deterministic	No	Yes	ITARA	CCSD	Yes	Illustrative example
Our study	*Railway infrastructure risk assessment*	*Yes*	*Picture fuzzy*	*Yes*	*Yes*	*Direct rating*	*THC entropy*	*Yes*	*Real-life*

Comparative analysis: CA; Correlation Coefficient and Standard Deviation: CCSD; Full Consistency Method: FUCOM; Group decision-making: GDM; Indifference Threshold-based Attribute Ratio Analysis: ITARA; Pivot Pairwise Relative Criteria Importance Assessment: PIPRECIA; Sensitivity analysis: SA; Tsallis–Havrda–Charvát: THC.

3. Preliminaries

This section provides some definitions of picture fuzzy sets.

Definition 1 [6,7]. *Let PFS A on a universe X is an object in the form of*:

$$\hat{A} = \left\{ < x,\ \mu_{\hat{A}}(x),\ \eta_{\hat{A}}(x),\ \upsilon_{\hat{A}}(x) > \middle| x \in X \right\}, \tag{1}$$

where $\mu_{\hat{A}}(x) \in [0,\ 1]$ *is called the degree of positive membership of x in* $\hat{A}$; $\eta_{\hat{A}}(x) \in [0,\ 1]$ *is the degree of neutral membership of x in* $\hat{A}$; $\upsilon_{\hat{A}} \in [0,\ 1]$ *is the degree of negative membership of x in* $\hat{A}$; *and* $\mu_{\hat{A}}(x)$, $\eta_{\hat{A}}(x)$, *and* $\upsilon_{\hat{A}}(x)$ satisfy the following condition:

$$0 \le \mu_{\hat{A}}(x) + \eta_{\hat{A}}(x) + \upsilon_{\hat{A}}(x) \le 1,\ \forall x \in X. \tag{2}$$

The degree of refusal membership of x in the PFS A can be calculated as follows:

$$\xi_{\hat{A}}(x) = 1 - (\mu_{\hat{A}}(x) + \eta_{\hat{A}}(x) + \upsilon_{\hat{A}}(x)),\ \forall x \in X. \tag{3}$$

If X has only one element, then $\hat{A} = \{< x,\ \mu_{\hat{A}}(x),\ \eta_{\hat{A}}(x),\ \upsilon_{\hat{A}}(x) > | x \in X\}$ *is called a picture fuzzy number (PFN) in which* $\mu_{\hat{A}}(x) \in [0,\ 1]$, $\eta_{\hat{A}}(x) \in [0,\ 1]$, $\upsilon_{\hat{A}} \in [0,\ 1]$, *and* $0 \le \mu_{\hat{A}}(x) + \eta_{\hat{A}}(x) + \upsilon_{\hat{A}}(x) \le 1$. *For convenience, a PFN is denoted by* $\hat{A} =< \mu_{\hat{A}},\ \eta_{\hat{A}},\ \upsilon_{\hat{A}} >$ *[64,65].*

Definition 2 [66]. As a generalization of the algebraic product and sum, and Einstein T-conorm and T-norm, the Hamacher T-conorm and T-norm is more general and more flexible. Let $\hat{A} =< \mu_{\hat{A}},\ \eta_{\hat{A}},\ \upsilon_{\hat{A}} >$, $\hat{A}_1 =< \mu_{\hat{A}_1},\ \eta_{\hat{A}_1},\ \upsilon_{\hat{A}_1} >$, *and* $\hat{A}_2 =< \mu_{\hat{A}_2},\ \eta_{\hat{A}_2},\ \upsilon_{\hat{A}_2} >$ be three PFNs, operational parameter $\zeta > 0$, and $\lambda > 0$. The Hamacher T-norm and T-conorm operations of PFNs are defined as follows:

(a) Addition "⊕"

$$\begin{aligned} \hat{A}_1 \oplus \hat{A}_2 = \ & < \frac{\mu_{\hat{A}_1} + \mu_{\hat{A}_2} - \mu_{\hat{A}_1}\mu_{\hat{A}_2} - (1-\zeta)\mu_{\hat{A}_1}\mu_{\hat{A}_2}}{1-(1-\zeta)\mu_{\hat{A}_1}\mu_{\hat{A}_2}}, \frac{\eta_{\hat{A}_1}\eta_{\hat{A}_2}}{\zeta+(1-\zeta)(\eta_{\hat{A}_1}+\eta_{\hat{A}_2}-\eta_{\hat{A}_1}\eta_{\hat{A}_2})}, \\ & \frac{\upsilon_{\hat{A}_1}\upsilon_{\hat{A}_2}}{\zeta+(1-\zeta)(\upsilon_{\hat{A}_1}+\upsilon_{\hat{A}_2}-\upsilon_{\hat{A}_1}\upsilon_{\hat{A}_2})} > , \end{aligned} \tag{4}$$

(b) Multiplication "⊗"

$$\begin{aligned} \hat{A}_1 \otimes \hat{A}_2 = \ & < \frac{\mu_{\hat{A}_1}\mu_{\hat{A}_2}}{\zeta+(1-\zeta)(\mu_{\hat{A}_1}+\mu_{\hat{A}_2}-\mu_{\hat{A}_1}\mu_{\hat{A}_2})}, \frac{\eta_{\hat{A}_1}+\eta_{\hat{A}_2}-\eta_{\hat{A}_1}\eta_{\hat{A}_2}-(1-\zeta)\eta_{\hat{A}_1}\eta_{\hat{A}_2}}{1-(1-\zeta)\eta_{\hat{A}_1}\eta_{\hat{A}_2}}, \\ & \frac{\upsilon_{\hat{A}_1}+\upsilon_{\hat{A}_2}-\upsilon_{\hat{A}_1}\upsilon_{\hat{A}_2}-(1-\zeta)\upsilon_{\hat{A}_1}\upsilon_{\hat{A}_2}}{1-(1-\zeta)\upsilon_{\hat{A}_1}\upsilon_{\hat{A}_2}} > , \end{aligned} \tag{5}$$

(c) Scalar multiplication

$$\begin{aligned} \lambda \cdot \hat{A} = \ & < \frac{(1+(\zeta-1)\mu_{\hat{A}})^{\lambda} - (1-\mu_{\hat{A}})^{\lambda}}{[1+(\zeta-1)\mu_{\hat{A}}]^{\lambda} + (\zeta-1)(1-\mu_{\hat{A}})^{\lambda}}, \frac{\zeta(\eta_{\hat{A}})^{\lambda}}{[1+(\zeta-1)(1-\eta_{\hat{A}})]^{\lambda} + (\zeta-1)(\eta_{\hat{A}})^{\lambda}}, \\ & \frac{\zeta(\upsilon_{\hat{A}})^{\lambda}}{[1+(\zeta-1)(1-\upsilon_{\hat{A}})]^{\lambda} + (\zeta-1)(\upsilon_{\hat{A}})^{\lambda}} > , \end{aligned} \tag{6}$$

(d) Power

$$\begin{aligned} \hat{A}^{\lambda} = \ & < \frac{\zeta(\mu_{\hat{A}})^{\lambda}}{[1+(\zeta-1)(1-\mu_{\hat{A}})]^{\lambda} + (\zeta-1)(\mu_{\hat{A}})^{\lambda}}, \frac{[1+(\zeta-1)\eta_{\hat{A}}]^{\lambda} - (1-\eta_{\hat{A}})^{\lambda}}{[1+(\zeta-1)\eta_{\hat{A}}]^{\lambda} + (\zeta-1)(1-\eta_{\hat{A}})^{\lambda}}, \\ & \frac{[1+(\zeta-1)\upsilon_{\hat{A}}]^{\lambda} - (1-\upsilon_{\hat{A}})^{\lambda}}{[1+(\zeta-1)\upsilon_{\hat{A}}]^{\lambda} + (\zeta-1)(1-\upsilon_{\hat{A}})^{\lambda}} > . \end{aligned} \tag{7}$$

Definition 3 [67]. *Let* $\hat{A} = \left\{\hat{A}_1, \ldots, \hat{A}_f\right\}$ *and* $\hat{B} = \left\{\hat{B}_1, \ldots, \hat{B}_f\right\}$ *be two PFSs in X. A function En: PFSs(X)* $\rightarrow$ *[0, 1] is an entropy on PFS, if En satisfies the following axiomatic requirements:*

(a) Sharpness: $En(\hat{A}) = 0$, *if and only if* $\hat{A}$ *is a crisp set.*

(b) Maximality: $En(\hat{A}) = 1$, *if* $\mu_{\hat{A}_t} = \eta_{\hat{A}_t} = \upsilon_{\hat{A}_t} = \xi_{\hat{A}_t} = 0.25$ *for all* $t = 1, \ldots, f$.

(c) Resolution: $En(\hat{A}) \leq En(\hat{B})$, *if* $\hat{A}$, $\hat{B} \in PFS(x)$ *satisfy either* $\mu_{\hat{A}_t} \leq \mu_{\hat{B}_t}$, $\eta_{\hat{A}_t} \leq \eta_{\hat{B}_t}$, $\upsilon_{\hat{A}_t} \leq \upsilon_{\hat{B}_t}$ *when* $\max\left\{\mu_{\hat{A}_t}, \eta_{\hat{A}_t}, \upsilon_{\hat{A}_t}\right\} \leq 0.25$ *or* $\mu_{\hat{A}_t} \geq \mu_{\hat{B}_t}$, $\eta_{\hat{A}_t} \geq \eta_{\hat{B}_t}$, $\upsilon_{\hat{A}_t} \geq \upsilon_{\hat{B}_t}$ *when* $\min\left\{\mu_{\hat{B}_t}, \eta_{\hat{B}_t}, \upsilon_{\hat{B}_t}\right\} \geq 0.25$ *for all* $t = 1, \ldots, f$.

(d) Symmetry: $En(\hat{A}) = En(\hat{A}^c)$, *where* $\hat{A}^c$ *denotes the complement of* $\hat{A}$.

Definition 4 [68]. *Tsallis–Havrda–Charvát entropy [69,70] is a generalized form of Shannon entropy [71]. This well-known one-parametric entropy is suitable for real-world applications, since its strength lies in properties and applications. Let* $\hat{A} = \left\{\hat{A}_1, \ldots, \hat{A}_f\right\}$ *be a PFS in X and* $\chi > 0$ $(\neq 1)$ *be the information measure parameter. The picture fuzzy Tsallis–Havrda–Charvát entropy measure of the PFS A is defined as follows:*

$$En^{\chi}_{THC}(A) = \frac{1}{f(1-\chi)} \sum_{t=1}^{f} \{[(\mu_{\hat{A}_t})^{\chi} + (\eta_{\hat{A}_t})^{\chi} + (\upsilon_{\hat{A}_t})^{\chi} + (\xi_{\hat{A}_t})^{\chi}] - 1\}, \ \chi \in (0, 1) \cup (1, \infty). \tag{8}$$

Definition 5 [66]. *Let* $\hat{A}_l = <\mu_{\hat{A}_l}, \eta_{\hat{A}_l}, \upsilon_{\hat{A}_l}>$ $(l = 1, \ldots, s)$ *be a collection of PFNs, the operational parameter* $\zeta > 0$, *and* $\varphi = (\varphi_1, \ldots, \varphi_s)^T$ *be the weight vector of the collection of PFNs with* $\varphi_l > 0$ *and* $\sum_{l=1}^{s} \varphi_l = 1$. *The picture fuzzy Hamacher weighted average (PFHWA) operator is defined as follows:*

$$\begin{aligned} PFHWA_{\varphi}(\hat{A}_1, \ldots, \hat{A}_s) = \bigoplus_{l=1}^{s} (\varphi_l \hat{A}_l) = < & \frac{\prod_{l=1}^{s} [1+(\zeta-1)\mu_{\hat{A}_l}]^{\varphi_l} - \prod_{l=1}^{s} (1-\mu_{\hat{A}_l})^{\varphi_l}}{\prod_{l=1}^{s} [1+(\zeta-1)\mu_{\hat{A}_l}]^{\varphi_l} + (\zeta-1)\prod_{l=1}^{s} (1-\mu_{\hat{A}_l})^{\varphi_l}}, \\ & \frac{\zeta \prod_{l=1}^{s} (\eta_{\hat{A}_l})^{\varphi_l}}{\prod_{l=1}^{s} [1+(\zeta-1)(1-\eta_{\hat{A}_l})]^{\varphi_l} + (\zeta-1)\prod_{l=1}^{s} (\eta_{\hat{A}_l})^{\varphi_l}}, \frac{\zeta \prod_{l=1}^{s} (\upsilon_{\hat{A}_l})^{\varphi_l}}{\prod_{l=1}^{s} [1+(\zeta-1)(1-\upsilon_{\hat{A}_l})]^{\varphi_l} + (\zeta-1)\prod_{l=1}^{s} (\upsilon_{\hat{A}_l})^{\varphi_l}} >, \end{aligned} \tag{9}$$

Definition 6 [72,73]. *Let* $\hat{A} = <\mu_{\hat{A}}, \eta_{\hat{A}}, \upsilon_{\hat{A}}>$ *be a PFN. A two-step defuzzification method to obtain a crisp value of the PFN* $\hat{A}$ is:

Step 1. Distribute the neutral degree to the positive and negative degrees as follows:

$$\mu'_{\hat{A}} = \mu_{\hat{A}} + \frac{\eta_{\hat{A}}}{2}, \tag{10}$$

$$\upsilon'_{\hat{A}} = \upsilon_{\hat{A}} + \frac{\eta_{\hat{A}}}{2}. \tag{11}$$

Step 2. Calculate the crisp value y by:

$$y = \mu'_{\hat{A}} + \frac{1 + \mu'_{\hat{A}} - \upsilon'_{\hat{A}}}{2} \xi_{\hat{A}}. \tag{12}$$

4. Picture Fuzzy Group MCDM Approach for Risk Assessment

This section presents the developed picture fuzzy group MCDM approach for risk assessment of railway infrastructure (Figure 1). The approach has three phases. In the first phase, linguistic importance evaluations towards railway infrastructures and risk factors are collected from invited experts and expressed as PFNs. In the second phase, subjective, objective, and hybrid importance of risk factors

are computed. This phase represents the formulated picture fuzzy hybrid method for risk factor prioritization. Firstly, the picture fuzzy direct rating method is utilized to compute the subjective importance of risk factors. Secondly, the picture fuzzy Tsallis–Havrda–Charvát entropy method is applied to calculate the objective importance of risk factors. Thirdly, the subjective and objective importance are hybridized. In the last phase, the formulated picture fuzzy MARCOS method is used to rank railway infrastructures.

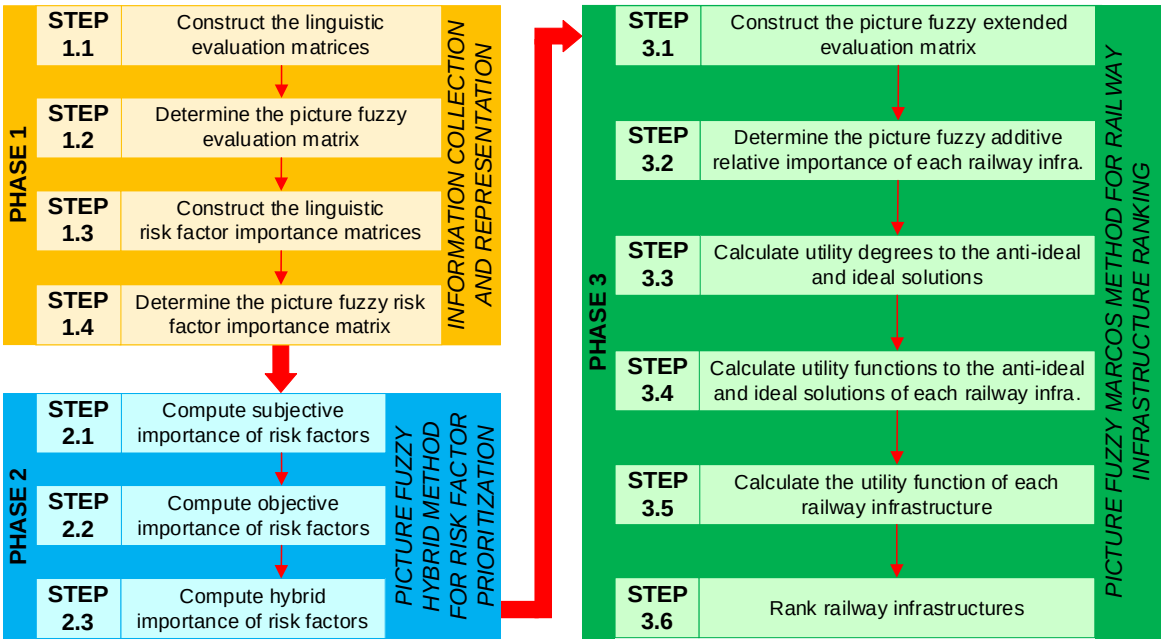

Figure 1. The flowchart of the picture fuzzy group MCDM approach for risk assessment of railway infrastructure.

Let A = $\{A_1, \ldots, A_m\}$ ($m \geq 2$) be a finite set of railway infrastructures that experts have to choose from, D = $\{D_1, \ldots, D_k\}$ ($k \geq 2$) be a set of invited experts, and C = $\{C_1, \ldots, C_n\}$ ($n \geq 2$) be a finite set of risk factors. The phases and encompassed steps of the developed group MCDM approach are given in the following:

Phase 1. *Information collection and representation.*

Step 1.1. Construct the linguistic evaluation matrices $\Gamma_i = [\gamma^i_{ej}]_{k\times n}$:

$$\Gamma_i = \begin{array}{c} \\ D_1 \\ \vdots \\ D_k \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} \gamma^i_{1\,1} & \cdots & \gamma^i_{1\,n} \\ \vdots & \ddots & \vdots \\ \gamma^i_{k\,1} & \cdots & \gamma^i_{k\,n} \end{bmatrix} \end{array}, \; i = 1, \ldots, m, \tag{13}$$

where m, k, and n are the number of railway infrastructures, invited experts, and risk factors, respectively; γ^i_{ej} *(*$i = 1, \ldots, m$*;* $e = 1, \ldots, k$*;* $j = 1, \ldots, n$*) is the linguistic evaluation (i.e., yes, abstain, no, or refusal) given by the invited expert* D_e *towards the railway infrastructure* A_i *with respect to the risk factor* C_j*.*

Step 1.2. Determine the picture fuzzy evaluation matrix $\hat{Z} = [\hat{z}_{ij}]_{m\times n}$:

$$\hat{Z} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} \hat{z}_{1\,1} & \cdots & \hat{z}_{1\,n} \\ \vdots & \ddots & \vdots \\ \hat{z}_{m\,1} & \cdots & \hat{z}_{m\,n} \end{bmatrix} \end{array}, \tag{14}$$

where $\hat{z}_{ij} =< \mu_{\hat{z}_{ij}}, \eta_{\hat{z}_{ij}}, \upsilon_{\hat{z}_{ij}} >$ *is a PFN, which represents an evaluation of the railway infrastructure* A_i *with respect to the risk factor* C_j *given by the invited experts. Importance evaluations are represented as PFNs by computing the share of each item (i.e., yes, abstain, no, or refusal) in the related voting results of the invited experts.*

Step 1.3. Construct the linguistic risk factor importance matrices $\Psi^e = [\psi_j^e]_{n\times 1}$:

$$\Psi^e = \begin{matrix} C_1 \\ \vdots \\ C_n \end{matrix} \begin{bmatrix} \psi_1^e \\ \vdots \\ \psi_n^e \end{bmatrix}, \; e = 1, \ldots, k, \tag{15}$$

where ψ_j^e *is the linguistic importance evaluation (i.e., yes, abstain, no, or refusal) given by the invited expert* D_e *towards the risk factor* C_j*.*

Step 1.4. Determine the picture fuzzy risk factor importance matrix $\hat{V} = [\hat{\upsilon}_j]_{n\times 1}$:

$$\hat{V} = \begin{matrix} C_1 \\ \vdots \\ C_n \end{matrix} \begin{bmatrix} \hat{\upsilon}_1 \\ \vdots \\ \hat{\upsilon}_n \end{bmatrix}, \tag{16}$$

where $\hat{\upsilon}_j < \mu_{\hat{\upsilon}_j}, \eta_{\hat{\upsilon}_j}, \upsilon_{\hat{\upsilon}_j} >$ *is a PFN, which represents the importance evaluation of the risk factor* C_j *given by the invited experts. It is computed as the share of each item in the corresponding voting results of the invited experts.*

Phase 2. *Picture fuzzy hybrid method for risk factor prioritization.*

Step 2.1. Compute subjective importance of risk factors:

$$w_j^S = \frac{\mu_{\hat{\upsilon}_j} + \frac{\eta_{\hat{\upsilon}_j}}{2} + \frac{\xi_{\hat{\upsilon}_j}}{2}(1 + \mu_{\hat{\upsilon}_j} - \upsilon_{\hat{\upsilon}_j})}{\sum\limits_{t=1}^{n} [\mu_{\hat{\upsilon}_t} + \frac{\eta_{\hat{\upsilon}_t}}{2} + \frac{\xi_{\hat{\upsilon}_t}}{2}(1 + \mu_{\hat{\upsilon}_t} - \upsilon_{\hat{\upsilon}_t})]}, \; j = 1, \ldots, n, \tag{17}$$

where $\hat{\upsilon}_j < \mu_{\hat{\upsilon}_j}, \eta_{\hat{\upsilon}_j}, \upsilon_{\hat{\upsilon}_j} >$ *is a PFN, which represents the importance evaluation of the risk factor* C_j *given by the invited experts; and* $w^S = (w_1^S, \ldots, w_n^S)^T$ *represents the subjective importance vector of the risk factors, with* $w_j^S \in [0, 1]$ *and* $\sum\limits_{j=1}^{n} w_j^S = 1$*. The subjective importance of risk factors are computed by using the picture fuzzy direct rating method.*

Step 2.2. Compute objective importance of risk factors:

$$w_j^O = \frac{1 - \frac{1}{m(1-\chi)} \sum\limits_{i=1}^{m} \{[(\mu_{\hat{z}_{ij}})^\chi + (\eta_{\hat{z}_{ij}})^\chi + (\upsilon_{\hat{z}_{ij}})^\chi + (\xi_{\hat{z}_{ij}})^\chi] - 1\}}{\sum\limits_{t=1}^{n} \{1 - \frac{1}{m(1-\chi)} \sum\limits_{l=1}^{m} \left\{[(\mu_{\hat{z}_{lt}})^\chi + (\eta_{\hat{z}_{lt}})^\chi + (\upsilon_{\hat{z}_{lt}})^\chi + (\xi_{\hat{z}_{lt}})^\chi] - 1\right\}}, \; \chi \in (0, 1) \cup (1, \infty); \; j = 1, \ldots, n, \tag{18}$$

where $\hat{z}_{ij} =< \mu_{\hat{z}_{ij}}, \eta_{\hat{z}_{ij}}, \upsilon_{\hat{z}_{ij}} >$ *is a PFN which represents an evaluation of the railway infrastructure* A_i *with respect to the risk factor* C_j *given by the experts;* χ *is the information measure parameter; and* $w^O = (w_1^O, \ldots, w_n^O)^T$ *represents the objective importance vector of the risk factors, with* $w_j^O \in [0, 1]$ *and* $\sum\limits_{j=1}^{n} w_j^O = 1$*. The objective importance of risk factors are computed by using the picture fuzzy Tsallis–Havrda–Charvát entropy method.*

Step 2.3. Compute hybrid importance of risk factors:

$$w_j = \gamma w_j^S + (1 - \gamma) w_j^O, \; j = 1, \ldots, n, \tag{19}$$

where $\gamma \in [0, 1]$ *represents the trade-off parameter;* $w = (w_1, \ldots, w_n)^T$ *is the hybrid importance vector of the risk factors, with* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$. *The risk factors have hybrid importance when* $\gamma \in (0, 1)$. *They have exclusively subjective and objective importance when* $\gamma = 1$ *and* $\gamma = 0$, *respectively.*

Phase 3. *Picture fuzzy MARCOS method for railway infrastructure ranking.*

Step 3.1. Construct the picture fuzzy extended evaluation matrix $\hat{\Phi} = [\hat{\phi}_{i'j}]_{(m+2)\times n}$:

$$\hat{\Phi} = \begin{array}{c} \\ A_0 \\ A_1 \\ \vdots \\ A_m \\ A_{m+1} \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} \hat{\phi}_{0\,1} & \cdots & \hat{\phi}_{0\,n} \\ \hat{\phi}_{1\,1} & \cdots & \hat{\phi}_{1\,n} \\ \vdots & \ddots & \vdots \\ \hat{\phi}_{m\,1} & \cdots & \hat{\phi}_{m\,n} \\ \hat{\phi}_{m+1\,1} & \cdots & \hat{\phi}_{m+1\,n} \end{bmatrix} \end{array}, \tag{20}$$

where $\mathrm{A}' = \{A_0, \ldots, A_{i'}, \ldots, A_{m+1}\}(i' = 0, \ldots, m+1)$ *is a finite set of railway infrastructures;* $\hat{\phi}_{ij} =< \mu_{\hat{\phi}_{ij}}, \eta_{\hat{\phi}_{ij}}, \upsilon_{\hat{\phi}_{ij}} >$ *is a PFN which represents an evaluation of the railway infrastructure* A_i *with respect to the risk factor* C_j *given by the experts:*

$$\hat{\phi}_{ij} = \hat{z}_{ij} =< \mu_{\hat{z}_{ij}}, \eta_{\hat{z}_{ij}}, \upsilon_{\hat{z}_{ij}} >, i = 1, \ldots, m; j = 1, \ldots, n, \tag{21}$$

In this step, the picture fuzzy evaluation matrix is extended by adding the anti-ideal and ideal railway infrastructures.

(1) Anti-ideal railway infrastructure $A_0 = \left\{\hat{\phi}_{0\,1}, \ldots, \hat{\phi}_{0\,n}\right\}$:

$$\hat{\phi}_{0\,j} =< \mu_{\hat{\phi}_{0\,j}}, \eta_{\hat{\phi}_{0\,j}}, \upsilon_{\hat{\phi}_{0\,j}} >=< \max_{i=1,\ldots,m} \mu_{\hat{z}_{ij}}, \min_{i=1,\ldots,m} \eta_{\hat{z}_{ij}}, \min_{i=1,\ldots,m} \upsilon_{\hat{z}_{ij}} >, j = 1, \ldots, n, \tag{22}$$

where $\hat{\phi}_{0\,j} =< \mu_{\hat{\phi}_{0\,j}}, \eta_{\hat{\phi}_{0\,j}}, \upsilon_{\hat{\phi}_{0\,j}} > (j = 1, \ldots, n)$ *is a collection of PFNs, which represent anti-ideal values for each risk factor.*

(2) Ideal railway infrastructure $A_{m+1} = \left\{\hat{\phi}_{m+1\,1}, \ldots, \hat{\phi}_{m+1\,n}\right\}$:

$$\hat{\phi}_{m+1\,j} =< \mu_{\hat{\phi}_{m+1\,j}}, \eta_{\hat{\phi}_{m+1\,j}}, \upsilon_{\hat{\phi}_{m+1\,j}} >=< \min_{i=1,\ldots,m} \mu_{\hat{z}_{ij}}, \min_{i=1,\ldots,m} \eta_{\hat{z}_{ij}}, \max_{i=1,\ldots,m} \upsilon_{\hat{z}_{ij}} >, j = 1, \ldots, n, \tag{23}$$

where $\hat{\phi}_{m+1\,j} =< \mu_{\hat{\phi}_{m+1\,j}}, \eta_{\hat{\phi}_{m+1\,j}}, \upsilon_{\hat{\phi}_{m+1\,j}} > (j = 1, \ldots, n)$ *is a collection of PFNs which represent ideal values for each risk factor.*

Step 3.2. Determine the picture fuzzy additive relative importance of each railway infrastructure as follows:

$$\begin{aligned} \hat{G}_{i'} &=< \mu_{\hat{G}_{i'}}, \eta_{\hat{G}_{i'}}, \upsilon_{\hat{G}_{i'}} >= PFHWA_w(\hat{\phi}_{i'1}, \ldots, \hat{\phi}_{i'n}) = \overset{n}{\underset{j=1}{\oplus}} (w_j \cdot \hat{\phi}_{i'j}) \\ &=< \frac{\prod_{j=1}^{n} [1+(\zeta-1)\mu_{\hat{\phi}_{i'j}}]^{w_j} - \prod_{j=1}^{n} (1-\mu_{\hat{\phi}_{i'j}})^{w_j}}{\prod_{j=1}^{n} [1+(\zeta-1)\mu_{\hat{\phi}_{i'j}}]^{w_j} + (\zeta-1)\prod_{j=1}^{n} (1-\mu_{\hat{\phi}_{i'j}})^{w_j}}, \frac{\zeta \prod_{l=1}^{n} (\eta_{\hat{\phi}_{i'j}})^{w_j}}{\prod_{j=1}^{n} [1+(\zeta-1)(1-\eta_{\hat{\phi}_{i'j}})]^{w_j} + (\zeta-1)\prod_{j=1}^{n} (\eta_{\hat{\phi}_{i'j}})^{w_j}}, \\ &\frac{\zeta \prod_{l=1}^{n} (\upsilon_{\hat{\phi}_{i'j}})^{\varphi_l}}{\prod_{j=1}^{n} [1+(\zeta-1)(1-\upsilon_{\hat{\phi}_{i'j}})]^{w_j} + (\zeta-1)\prod_{j=1}^{n} (\upsilon_{\hat{\phi}_{i'j}})^{w_j}} >, i' = 0, \ldots, m+1; \zeta > 0. \end{aligned} \tag{24}$$

where ζ *is the operational parameter.*

Step 3.3. Calculate utility degrees to the anti-ideal and ideal solutions of each railway infrastructure.

(1) Utility degree to the anti-ideal solution:

$$K_{i'}^{-} = \frac{\mu_{\hat{G}_{i'}} + \frac{\eta_{\hat{G}_{i'}}}{2} + \frac{\xi_{\hat{G}_{i'}}}{2}(1 + \mu_{\hat{G}_{i'}} - v_{\hat{G}_{i'}})}{\mu_{\hat{G}_0} + \frac{\eta_{\hat{G}_0}}{2} + \frac{\xi_{\hat{G}_0}}{2}(1 + \mu_{\hat{G}_0} - v_{\hat{G}_0})}, \; i' = 0, \ldots, m+1, \tag{25}$$

(2) Utility degree to the ideal solution:

$$K_{i'}^{+} = \frac{\mu_{\hat{G}_{i'}} + \frac{\eta_{\hat{G}_{i'}}}{2} + \frac{\xi_{\hat{G}_{i'}}}{2}(1 + \mu_{\hat{G}_{i'}} - v_{\hat{G}_{i'}})}{\mu_{\hat{G}_{m+1}} + \frac{\eta_{\hat{G}_{m+1}}}{2} + \frac{\xi_{\hat{G}_{m+1}}}{2}(1 + \mu_{\hat{G}_{m+1}} - v_{\hat{G}_{m+1}})}, \; i' = 0, \ldots, m+1. \tag{26}$$

Step 3.4. Calculate utility functions to the anti-ideal and ideal solutions.
(1) Utility function to the anti-ideal solution:

$$f(K^{-}) = \frac{K_0^{+}}{K_0^{-} + K_0^{+}}, \tag{27}$$

(2) Utility function to the ideal solution:

$$f(K^{+}) = \frac{K_{m+1}^{-}}{K_{m+1}^{-} + K_{m+1}^{+}}. \tag{28}$$

Step 3.5. Calculate the utility function of each railway infrastructure as follows:

$$f(K_i) = \frac{(K_i^{-} + K_i^{+})[f(K^{+}) \cdot f(K^{-})]}{f(K^{-}) + f(K^{+}) - f(K^{-}) \cdot f(K^{+})}, \; i = 1, \ldots, m, \tag{29}$$

Step 3.6. Rank railway infrastructures.
Rank the railway infrastructures according to the increasing values of utility functions. The lowest value is the safest railway infrastructure.

5. Case Study

According to the 2020 census, the Czech Republic has 10.7 million inhabitants. More than 50% of the population use railway transportation. A real-life case study of risk assessment of railway infrastructure in the Czech Republic context is presented to evaluate railway infrastructure safety and demonstrate the utility of the formulated approach. The approach is used to prioritize twenty-three relevant risk factors (Table 3) and assess six railway infrastructures (Figure 2).

The investigated railway infrastructures in the Czech Republic context are:

- *(A_1) Pardubice* (Figure 3a). The railway infrastructure of Pardubice is an important network hub. It is served by international and local trains. This railway infrastructure has connections to Prague, Brno, Bratislava, Budapest, and other large cities. The main railway station, operated by Czech Railways, is vital for passenger traffic and freight transport. Its passenger hall became a place where homeless people concentrate. This railway infrastructure was bombed several times during World War II and many reconstruction projects had been done. Additionally, few significant accidents occurred in the past.
- *(A_2) Hradec Kralove* (Figure 3b). It is well-connected with Pardubice. These two railway infrastructures are 21 km away. Hradec Kralove is also an important hub for both passengers and freight. Several significant accidents occurred. Since lots of people pass through its station, the risk of a terroristic attack is possible as well.

- (A_3) *Kolin* (Figure 3c). This railway infrastructure is located between the capital of the Czech Republic Prague and Pardubice. It is of huge importance for railway transportation in the Czech Republic as a whole, due to a high frequency of trains on the way to Prague. Several years ago, on the railway infrastructure in Kolin, illegal driving was noticed, but luckily there were no injuries and damage declared.
- (A_4) *Usti nad Labem* (Figure 3d). It is located near the border with Germany. This railway infrastructure counts many departures and arrivals of the local and international trains. Its railway station has been operable for more than sixty years. Due to its location, this railway infrastructure is exposed to various risks such as epidemics, terroristic attacks, and similar accidental events. In the past, some traffic accidents happened. For instance, a freight train passed a signal prohibiting driving and entered the built train path for a departing passenger train running along the adjacent track in the same direction. The drivers of both trains stopped in front of the switch, with the sides of the trains approaching each other less than a meter.
- (A_5) *Chomutov* (Figure 3e). The railway infrastructure in Chomutov is located in the north-west part of the Czech Republic. It is 86 km away from Prague and 50 km away from Usti nad Labem. Recently, this railway infrastructure had several significant accidents. In 2010, a freight train knocked down a woman and seriously injured her. In 2016, two people died after being hit by a freight train. The accident occurred at a railway crossing secured by a light warning signal without barriers. According to the information of the Railway Inspectorate, it was in operation at the time of the accident.
- (A_6) *Jaromer* (Figure 3f). This railway infrastructure connects Hradec Kralove with Pardubice. Some causes of accidents that happened in the past are walking on the track, passing a signal prohibiting driving, derailment of a train on the ground that was washed up on the track after torrential rain, and track defects.

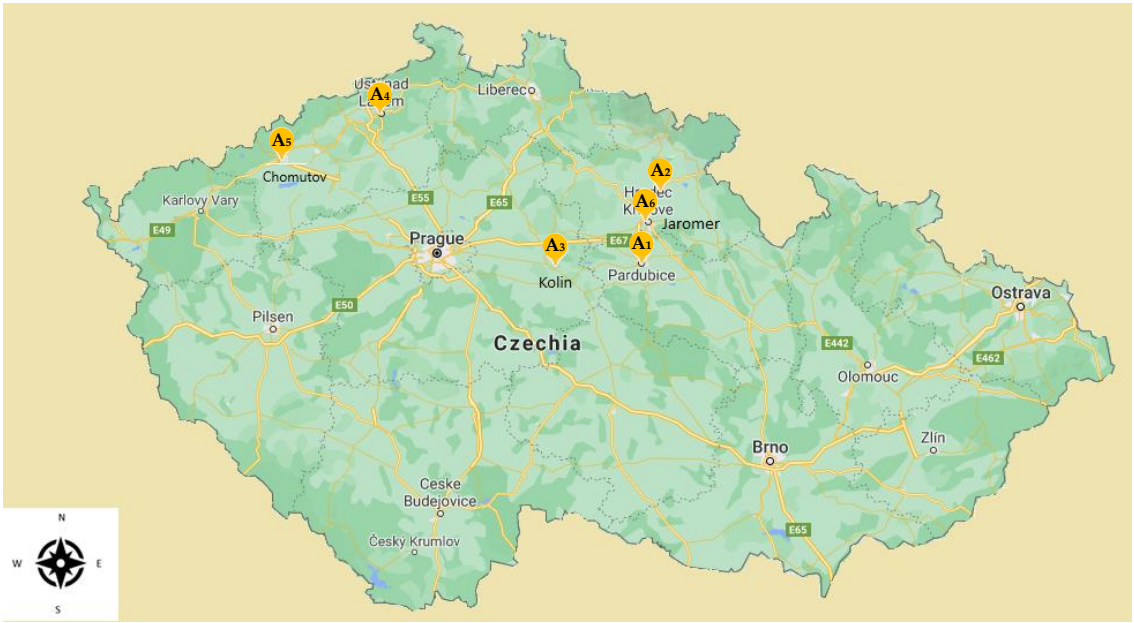

Figure 2. Locations of six assessed railway infrastructures in the Czech Republic.

Table 3. Risk factors influencing railway infrastructure in the Czech Republic.

Risk Factor	Code	Description
Prolonged drought	C_1	Prolonged drought is an accidental natural phenomenon caused by a deficit of precipitation. It causes significant economic damage and endangers public health, harms forests and crops, and increases the risk of fires.
Extreme temperatures	C_2	Extremely high temperatures are a phenomenon that can occur in the Czech Republic in the period from June to August, rarely in late May and early September. The daily maximum air temperature exceeds 30 °C for several days.
Flood	C_3	Temporary significant increase in the level of watercourses or other surface waters, during which water floods the area outside the riverbed and can cause damage. The Czech Republic has a dense hydrographic network with a length of 85.000 km.
Heavy rainfall	C_4	The occurrence of intense rainfall in urban areas results in exceeding the capacity of the sewer network, flooding the lower floors of buildings, and technical infrastructure with surface runoff rainwater.
Extreme wind	C_5	Wind speed in gusts greater than 30 m/s or in high gusts greater than 25 m/s. Its effects complicate the functionality of critical infrastructure entities. The average annual wind speed in the Czech Republic is between 2 and 4 m/s.
Epidemics	C_6	Mass human infection as a result of which, in a local and temporal context, the morbidity of an infectious disease increases above the usual limit. In the Czech Republic, epidemics occur sporadically.
Epiphytia	C_7	Mass plant disease is a spread of pests at a level that cannot be managed by conventional methods and means of protection. There is a risk of significant societal losses.
Epizootics	C_8	Mass animal disease occurs when emergency measures cannot be implemented by veterinary authorities. There is a risk of human infection with zoonoses.
Disrupted food supplies	C_9	Large-scale disruption of food supplies. It can be a secondary consequence of other emergencies.
Disrupted communication systems	C_{10}	Disrupted functionality of important electronic communication networks, which consist of broadcasting systems, routing equipment, and other means enabling the transmission of signals. It affects the monitoring systems and information support.
Critical information infrastructure violation	C_{11}	Planned attack or unintentional failure of technology and people, which leads to failure of services provided by information and communication systems, causes security violations of critical information infrastructure.
Special flood	C_{12}	A special flood is caused by a failure or accident through the rupture of a dam of a waterworks swelling or accumulating water or by an emergency solution of a critical situation.
Hazardous chemical leakage	C_{13}	Hazardous chemicals and mixtures are a source of risk for the occurrence of serious accidents. Accidents are manifested by the release of a chemical or mixture under uncontrolled conditions in a stationary installation. Physiochemical effects are fire and explosion. Biological effects are toxicity and ecotoxicity.
Disrupted water supply	C_{14}	Large-scale disruption of drinking water supplies for drinking, cooking, food, and beverage preparation.
Disrupted gas supply	C_{15}	Large-scale disruption of gas supplies. Equipment for gas extraction, transportation, or storage poses a local risk of explosion. The failure of cross-border points does not have an impact on the gas supply of the Czech Republic.
Disrupted oil supplies	C_{16}	A significant and sudden decrease in crude oil and petroleum supplies. This security threat impacts the economy and living conditions.
Radiation accidents	C_{17}	The radiation emergency is unmanageable with the forces and means of staff. It is manifested by the leakage of radioactive substances and requires the introduction of urgent protective measures for the population.
Disrupted electricity supply	C_{18}	Large-scale disruption of electricity supply. The significant and sudden shortage of electricity, which endangers the integrity, safety, and reliability of an electricity system. Transmission and distribution system operators implement measures and activities to prevent a state of emergency.
Migration wave	C_{19}	Foreigners from third countries coming to the state borders with the aim of legally or illegally enter this territory, transit through to other states, or reside in it. A large-scale migration wave is the arrival of about 750 foreigners/day or 5000 foreigners/month.
Legality violation	C_{20}	Large-scale legality violation is dangerous and intentional behavior in which there is a violation of legislation and security of the Czech Republic to endanger the lives and health of citizens, infrastructure, environment, and democratic system.
Economic destabilization	C_{21}	Large-scale economic destabilization, macroeconomic imbalances, reduction of economic performances, external vulnerability, and economic crisis threats to the financing of social and health expenditure.
Military assault	C_{22}	Violent action or direct attack on the territory of the Czech Republic by the military and other forces of a foreign power.
Human factor failure	C_{23}	Possible risk caused by human mistakes.

Figure 3. *Cont.*

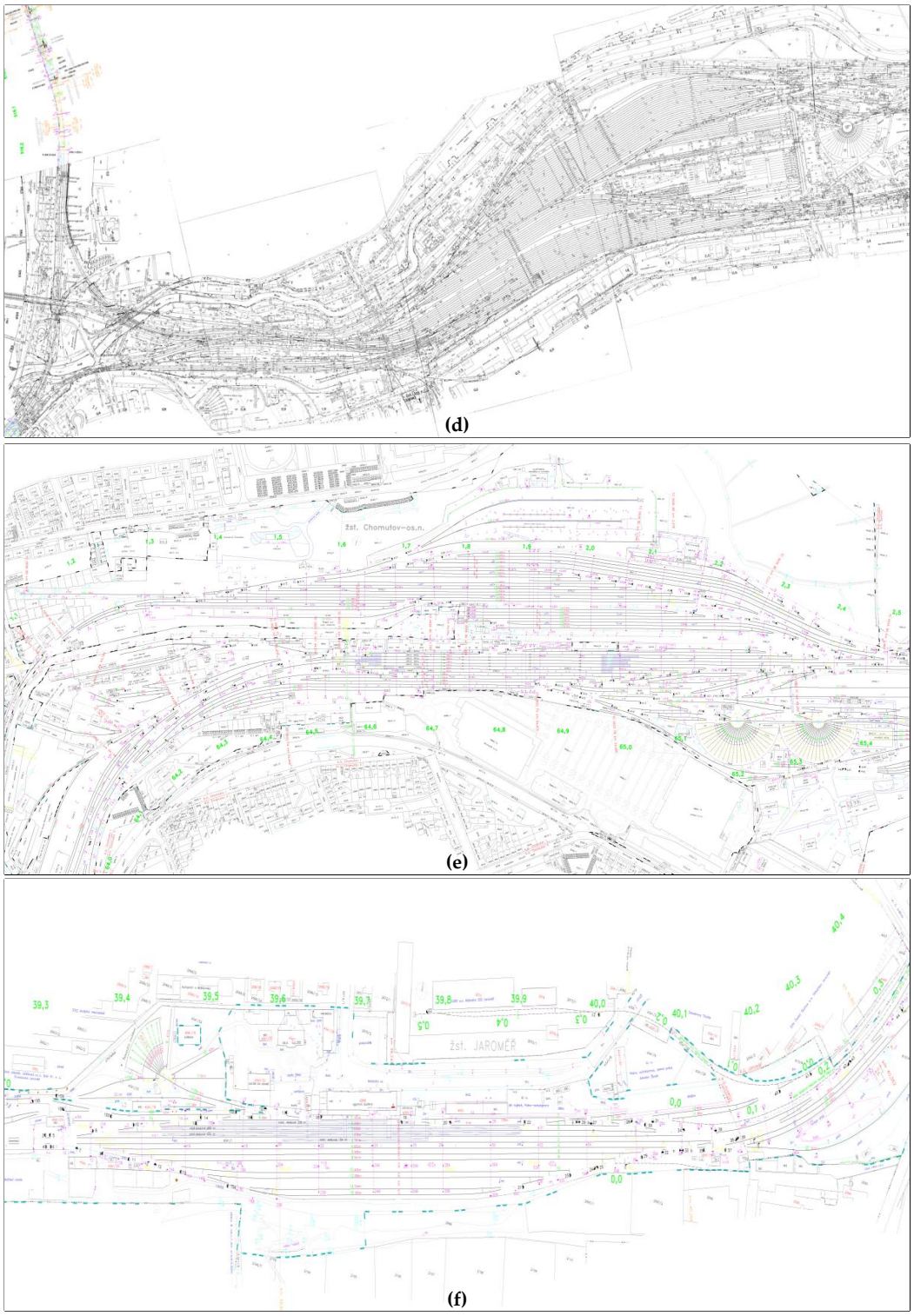

Figure 3. The railway infrastructures: (**a**) Pardubice; (**b**) Hradec Kralove; (**c**) Kolin; (**d**) Usti nad Labem; (**e**) Chomutov; (**f**) Jaromer.

A systematic approach is carried out to identify relevant risk factors influencing railway infrastructure in the Czech Republic from the literature [74,75]. Risk factors that do not apply to the Czech Republic context are not listed in Table 3. Each risk is briefly described in Table 3.

6. Results and Discussion

6.1. Experimental Results

Six assessed railway infrastructures in the Czech Republic context are "Pardubice" (A_1), "Hradec Kralove" (A_2), "Kolin" (A_3), "Usti nad Labem" (A_4), "Chomutov" (A_5), and "Jaromer" (A_6). Twenty-three risk factors influencing railway infrastructure are considered (Table 3). Ten railway planners participated in the case study (Table 4).

Table 4. The information about the invited railway planners.

Expert	Occupation	Qualifications	Experience (years)	Gender
D_1	Academia	Ph.D.	25	Male
D_2	Industry	Eng.	5	Male
D_3	Industry	Eng.	2	Male
D_4	Industry	Eng.	50	Male
D_5	Industry	Ph.D.	13	Male
D_6	Academia	Ph.D.	20	Female
D_7	Industry	M.Sc.	42	Female
D_8	Industry	B.Sc.	28	Female
D_9	Industry	Eng.	30	Female
D_{10}	Industry	Eng.	20	Male

The online questionnaire approach via Google Forms was utilized to collect evaluations of risk factors and railway infrastructures due to the COVID-19 outbreak. In the first part of the online questionnaire, the invited railway planners are asked to enter their e-mail address, name, occupation, qualifications, total years of experience, and gender. In the second part, the experts are asked to evaluate twenty-three risk factors influencing railway infrastructure in the Czech Republic. In the last part, the participants are requested to provide linguistic evaluations of six investigated railway infrastructures with respect to the risk factors.

Phase 1. *Information collection and representation.*

Step 1.1. The invited railway planners provided evaluations of six railway infrastructures in the Czech Republic with respect to the risk factors via the online questionnaire. Importance evaluations can be yes, abstain, no, and refusal. Group refusal of voting either is invalid voting papers or does not take the vote. As a result, six linguistic evaluation matrices are constructed with the help of Equation (13). Linguistic evaluations given by 10 relevant railway planners towards the railway infrastructures with respect to twenty-three risk factors are presented in Table 5.

Step 1.2. A picture fuzzy evaluation of railway infrastructure with respect to a risk factor is calculated as the proportion of each out of four items (i.e., yes, abstain, no, or refusal) in the corresponding online voting results of 10 questioned railway planners. The picture fuzzy evaluation matrix is given in Table 6. It is determined based on six linguistic evaluation matrices (Table 5) by using Equation (14).

Step 1.3. Twenty-three risk factors influencing railway infrastructure are evaluated by the relevant experts (Table 7). A sample of the online questionnaire to determine the importance of the risk factors is given in the Appendix A. Ten linguistic risk factor importance matrices are constructed by using Equation 15.

Step 1.4. A picture fuzzy evaluation of a risk factor is computed as the proportion of each out of four items in the corresponding results of the online questionnaire. The picture fuzzy risk factor importance matrix is provided in Table 8. It is determined based on ten linguistic risk factor importance matrices (Table 7) with the help of Equation (16).

Phase 2. *Picture fuzzy hybrid method for risk factor prioritization.*

Step 2.1. The subjective importance and ranks of twenty-three risk factors are given in Table 9. The values are determined based on the railway planners' voting on the risk factors influencing railway infrastructure (Table 8) by using the picture fuzzy direct rating method defined in Equation (17).

Step 2.2. The objective importance and ranks of the risk factors are presented in Table 9. The values are calculated based on the railway planners' voting on six railway infrastructures (Table 6) by using the picture fuzzy Tsallis–Havrda–Charvát entropy method defined in Equation (18). In the base case scenario, it is adopted that the information measure parameter χ of the developed picture fuzzy group MCDM approach for risk assessment of railway infrastructure is 1.4.

Step 2.3. The hybrid importance and ranks of the risk factors influencing railway infrastructure are provided in Table 9. The values are computed by using Equation (19). In the base case scenario, it is assumed that the value of the trade-off parameter γ of the formulated picture fuzzy group MCDM approach is 0.5. This value equally appraises the picture fuzzy direct rating method and the picture fuzzy Tsallis–Havrda–Charvát entropy method.

Table 5. Linguistic evaluations of the railway infrastructures.

RI	Expert	Risk Factor																						
		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}	C_{21}	C_{22}	C_{23}
A_1	D_1	N	A	N	A	N	Y	N	N	A	Y	Y	N	Y	N	N	A	N	A	A	Y	N	N	Y
	D_2	N	A	A	A	A	Y	N	N	N	Y	Y	N	Y	N	A	Y	A	Y	N	A	N	Y	Y
	D_3	N	A	A	N	A	A	N	N	N	Y	Y	N	A	N	N	Y	N	Y	N	Y	N	Y	Y
	D_4	N	Y	A	A	A	A	N	N	N	A	A	N	Y	N	N	N	N	A	N	N	N	N	A
	D_5	N	Y	Y	A	Y	A	N	N	N	Y	Y	N	Y	N	N	Y	Y	Y	A	Y	A	Y	Y
	D_6	N	Y	A	A	Y	A	N	N	A	A	Y	N	A	A	N	Y	R	Y	N	A	N	N	A
	D_7	R	N	A	N	N	N	R	R	R	N	N	A	Y	N	N	Y	N	A	N	A	R	A	Y
	D_8	N	A	A	A	A	A	A	N	A	A	A	N	N	N	N	Y	Y	N	N	A	N	Y	N
	D_9	N	Y	A	A	A	Y	N	A	N	Y	Y	N	A	N	A	Y	A	A	N	A	N	Y	Y
	D_{10}	A	Y	A	A	A	Y	N	N	N	Y	Y	N	A	A	N	Y	A	Y	N	A	N	Y	Y
A_2	D_1	N	A	N	A	N	A	N	N	A	N	A	N	N	A	N	N	N	A	A	A	A	N	Y
	D_2	N	N	A	A	A	Y	N	N	N	Y	Y	A	A	N	N	Y	A	Y	N	N	N	A	Y
	D_3	N	A	A	N	A	A	N	N	N	A	Y	N	N	N	N	A	N	Y	N	A	N	Y	Y
	D_4	N	Y	Y	Y	Y	A	N	N	N	A	A	N	N	N	N	N	N	A	N	N	N	N	N
	D_5	N	Y	Y	A	Y	A	N	N	N	Y	Y	Y	Y	N	N	Y	Y	Y	A	Y	A	Y	Y
	D_6	N	Y	A	A	Y	A	N	N	A	A	A	N	N	A	N	Y	R	Y	N	A	N	N	A
	D_7	R	N	N	N	N	Y	R	R	N	N	N	N	N	N	N	N	N	Y	N	A	N	A	Y
	D_8	A	N	A	N	N	Y	A	N	N	Y	Y	A	A	N	A	Y	Y	N	N	N	N	A	Y
	D_9	N	A	A	N	N	N	N	A	A	N	Y	A	A	N	N	Y	A	Y	A	A	N	A	Y
	D_{10}	N	N	A	A	Y	A	N	N	A	N	A	N	N	N	N	Y	A	Y	N	N	N	A	N
A_3	D_1	N	A	Y	A	N	A	N	N	A	N	A	N	Y	N	N	N	N	A	A	Y	N	N	A
	D_2	N	N	A	N	A	Y	N	N	N	Y	Y	N	Y	N	N	Y	A	Y	N	A	N	Y	Y
	D_3	N	A	A	N	A	A	N	N	N	A	Y	N	A	N	N	A	N	A	N	A	N	Y	Y
	D_4	N	Y	A	A	Y	A	N	N	N	A	Y	N	Y	N	N	N	A	Y	N	N	N	N	A
	D_5	N	Y	A	A	Y	A	N	N	N	Y	Y	A	Y	N	N	Y	Y	Y	Y	Y	A	Y	Y
	D_6	N	Y	A	A	Y	A	N	N	A	A	Y	N	A	A	N	Y	R	Y	N	A	N	N	A
	D_7	R	N	A	N	N	Y	R	A	N	N	N	A	Y	N	N	Y	N	Y	N	A	N	A	N
	D_8	N	N	N	N	N	N	A	N	N	A	Y	N	N	N	N	Y	A	A	N	A	N	Y	Y
	D_9	A	Y	A	N	A	Y	N	N	N	Y	Y	N	Y	N	A	Y	N	A	N	Y	N	Y	Y
	D_{10}	N	Y	A	N	A	Y	N	N	A	N	Y	N	A	N	A	A	A	N	N	A	N	Y	Y
A_4	D_1	N	A	Y	Y	N	A	N	N	Y	A	A	Y	Y	N	N	N	N	A	A	A	N	N	Y
	D_2	N	A	Y	A	Y	Y	N	N	N	Y	Y	A	Y	N	N	Y	A	Y	N	A	N	Y	Y
	D_3	N	N	A	N	A	N	N	N	N	A	Y	A	A	N	N	A	N	Y	N	A	N	Y	Y
	D_4	N	Y	Y	Y	Y	A	N	N	N	A	A	N	Y	N	N	N	N	A	N	N	N	N	A
	D_5	N	Y	A	Y	Y	A	N	N	N	Y	Y	A	Y	N	N	Y	Y	Y	A	Y	A	Y	Y
	D_6	N	Y	A	A	A	A	N	N	A	A	Y	N	A	A	N	Y	R	Y	N	A	N	N	A
	D_7	R	R	N	A	A	Y	R	R	N	N	N	A	A	N	N	Y	N	Y	N	A	N	A	Y
	D_8	N	Y	A	A	Y	Y	N	N	A	A	Y	N	N	N	N	Y	N	N	N	A	N	Y	N
	D_9	A	A	Y	Y	Y	Y	A	N	N	Y	Y	A	A	N	A	Y	A	Y	N	A	N	Y	Y
	D_{10}	N	A	Y	A	Y	A	Y	A	N	Y	Y	A	Y	A	N	Y	R	Y	N	A	N	Y	A
A_5	D_1	N	A	A	Y	A	A	N	N	N	N	N	N	A	N	N	N	N	N	A	A	N	N	Y
	D_2	N	N	N	N	A	Y	N	N	N	Y	Y	N	A	N	N	Y	A	Y	N	N	N	A	Y
	D_3	N	N	A	N	A	N	N	N	N	A	Y	Y	A	N	N	N	N	Y	N	A	N	Y	Y
	D_4	N	Y	A	A	Y	A	N	N	N	A	A	N	N	N	N	N	N	N	N	N	N	N	N
	D_5	N	A	N	N	Y	A	N	N	N	Y	Y	N	Y	N	N	A	A	Y	A	Y	A	Y	Y
	D_6	N	Y	N	A	A	A	N	N	A	A	Y	N	A	A	N	Y	R	Y	N	A	N	N	A
	D_7	R	R	N	N	N	Y	R	R	N	N	N	N	N	N	N	Y	N	Y	N	N	N	A	Y
	D_8	A	A	A	N	A	Y	A	A	N	Y	Y	A	A	N	N	Y	A	Y	N	Y	A	Y	Y
	D_9	N	Y	N	N	A	Y	Y	R	Y	Y	Y	N	A	N	N	Y	R	A	Y	Y	Y	A	Y
	D_{10}	N	N	N	N	A	Y	N	N	N	A	Y	Y	A	N	A	Y	R	Y	N	N	N	A	Y
A_6	D_1	N	N	N	A	A	A	N	N	A	N	N	N	N	N	N	N	N	N	A	N	N	N	Y
	D_2	N	N	N	A	A	Y	N	N	N	Y	Y	N	A	N	N	N	A	Y	N	N	N	N	Y
	D_3	N	N	A	N	A	N	N	N	N	A	Y	A	N	N	N	A	N	A	N	A	N	Y	Y
	D_4	N	Y	A	A	Y	A	N	N	N	N	A	A	N	N	N	N	N	N	N	N	N	N	N
	D_5	N	Y	A	A	Y	A	N	N	N	Y	Y	N	Y	N	N	A	Y	Y	A	Y	A	Y	Y
	D_6	N	Y	N	A	Y	A	N	N	A	A	Y	N	N	A	N	Y	R	Y	N	N	N	N	A
	D_7	R	A	N	N	N	Y	R	R	N	N	N	A	N	N	A	A	N	Y	N	N	N	A	A
	D_8	N	N	N	A	Y	A	A	A	N	N	Y	N	A	N	A	N	N	Y	N	N	N	N	Y
	D_9	A	Y	N	A	A	A	Y	A	N	Y	Y	N	A	N	A	N	N	Y	N	A	N	Y	Y
	D_{10}	R	N	N	A	A	A	N	N	N	Y	A	A	A	N	N	N	N	Y	N	N	N	N	Y

Abstain: A; No: N; Railway infrastructure: RI; Refusal: R; Yes: Y.

Table 6. The picture fuzzy evaluation matrix.

Risk Factor	Railway Infrastructure					
	A_1	A_2	A_3	A_4	A_5	A_6
C_1	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0, 0.1, 0.7>
C_2	<0.5, 0.4, 0.1>	<0.3, 0.3, 0.4>	<0.5, 0.2, 0.3>	<0.4, 0.4, 0.1>	<0.3, 0.3, 0.3>	<0.4, 0.1, 0.5>
C_3	<0.1, 0.8, 0.1>	<0.2, 0.6, 0.2>	<0.1, 0.8, 0.1>	<0.5, 0.4, 0.1>	<0, 0.4, 0.6>	<0, 0.3, 0.7>
C_4	<0, 0.8, 0.2>	<0.1, 0.5, 0.4>	<0, 0.4, 0.6>	<0.4, 0.5, 0.1>	<0.1, 0.2, 0.7>	<0, 0.8, 0.2>
C_5	<0.2, 0.6, 0.2>	<0.4, 0.2, 0.4>	<0.3, 0.4, 0.3>	<0.6, 0.3, 0.1>	<0.2, 0.7, 0.1>	<0.4, 0.5, 0.1>
C_6	<0.4, 0.5, 0.1>	<0.3, 0.6, 0.1>	<0.4, 0.5, 0.1>	<0.4, 0.5, 0.1>	<0.5, 0.4, 0.1>	<0.2, 0.7, 0.1>
C_7	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0.1, 0.1, 0.7>	<0.1, 0.1, 0.7>	<0.1, 0.1, 0.7>
C_8	<0, 0.2, 0.7>	<0, 0.1, 0.8>	<0, 0.1, 0.9>	<0, 0.1, 0.8>	<0, 0.1, 0.7>	<0, 0.2, 0.7>
C_9	<0, 0.2, 0.7>	<0, 0.4, 0.6>	<0, 0.3, 0.7>	<0.1, 0.2, 0.7>	<0.1, 0.1, 0.8>	<0, 0.2, 0.8>
C_{10}	<0.6, 0.3, 0.1>	<0.3, 0.3, 0.4>	<0.3, 0.4, 0.3>	<0.4, 0.5, 0.1>	<0.4, 0.4, 0.2>	<0.4, 0.2, 0.4>
C_{11}	<0.7, 0.2, 0.1>	<0.5, 0.4, 0.1>	<0.8, 0.1, 0.1>	<0.7, 0.2, 0.1>	<0.7, 0.1, 0.2>	<0.6, 0.2, 0.2>
C_{12}	<0, 0.1, 0.9>	<0.1, 0.3, 0.6>	<0, 0.2, 0.8>	<0.1, 0.6, 0.3>	<0.2, 0.1, 0.7>	<0, 0.4, 0.6>
C_{13}	<0.5, 0.4, 0.1>	<0.1, 0.3, 0.6>	<0.6, 0.3, 0.1>	<0.5, 0.4, 0.1>	<0.1, 0.7, 0.2>	<0.1, 0.4, 0.5>
C_{14}	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.1, 0.9>	<0, 0.2, 0.8>	<0, 0.1, 0.9>	<0, 0.1, 0.9>
C_{15}	<0, 0.2, 0.8>	<0, 0.1, 0.9>	<0, 0.2, 0.8>	<0, 0.1, 0.9>	<0, 0.1, 0.9>	<0, 0.3, 0.7>
C_{16}	<0.8, 0.1, 0.1>	<0.6, 0.1, 0.3>	<0.6, 0.2, 0.2>	<0.7, 0.1, 0.2>	<0.6, 0.1, 0.3>	<0.1, 0.3, 0.6>
C_{17}	<0.2, 0.3, 0.4>	<0.2, 0.3, 0.4>	<0.1, 0.4, 0.4>	<0.1, 0.2, 0.5>	<0, 0.3, 0.4>	<0.1, 0.1, 0.7>
C_{18}	<0.5, 0.4, 0.1>	<0.7, 0.2, 0.1>	<0.5, 0.4, 0.1>	<0.7, 0.2, 0.1>	<0.7, 0.1, 0.2>	<0.7, 0.1, 0.2>
C_{19}	<0, 0.2, 0.8>	<0, 0.3, 0.7>	<0.1, 0.1, 0.8>	<0, 0.2, 0.8>	<0.1, 0.2, 0.7>	<0, 0.2, 0.8>
C_{20}	<0.3, 0.6, 0.1>	<0.1, 0.5, 0.4>	<0.3, 0.6, 0.1>	<0.1, 0.8, 0.1>	<0.3, 0.3, 0.4>	<0.1, 0.2, 0.7>
C_{21}	<0, 0.1, 0.8>	<0, 0.2, 0.8>	<0, 0.1, 0.9>	<0, 0.1, 0.9>	<0.1, 0.2, 0.7>	<0, 0.1, 0.9>
C_{22}	<0.6, 0.1, 0.3>	<0.2, 0.5, 0.3>	<0.6, 0.1, 0.3>	<0.6, 0.1, 0.3>	<0.3, 0.4, 0.3>	<0.3, 0.1, 0.6>
C_{23}	<0.7, 0.2, 0.1>	<0.7, 0.1, 0.2>	<0.6, 0.3, 0.1>	<0.6, 0.3, 0.1>	<0.8, 0.1, 0.1>	<0.7, 0.2, 0.1>

Table 7. Linguistic importance evaluations of the risk factors.

Expert	Risk Factor																						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}	C_{21}	C_{22}	C_{23}
D_1	N	Y	Y	Y	Y	Y	N	N	Y	Y	Y	Y	A	A	A	Y	Y	Y	A	Y	A	Y	A
D_2	R	Y	Y	A	Y	Y	N	N	A	Y	Y	Y	Y	A	A	Y	Y	Y	A	Y	A	Y	Y
D_3	N	A	Y	N	A	A	N	N	N	A	Y	A	N	N	A	Y	A	Y	A	Y	N	Y	Y
D_4	N	Y	Y	A	Y	Y	N	N	A	Y	Y	A	A	N	N	Y	Y	Y	A	A	A	Y	Y
D_5	N	Y	Y	Y	Y	Y	N	N	N	Y	Y	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y
D_6	N	Y	Y	Y	Y	Y	N	N	A	Y	Y	Y	Y	A	N	Y	Y	Y	A	Y	A	Y	Y
D_7	R	Y	Y	Y	A	Y	R	R	R	A	N	A	A	N	N	Y	A	Y	N	Y	A	Y	Y
D_8	N	A	Y	A	Y	Y	N	N	N	Y	Y	Y	A	N	N	Y	Y	Y	Y	Y	Y	Y	Y
D_9	R	Y	Y	A	Y	Y	N	N	A	A	N	A	Y	A	N	Y	Y	Y	A	Y	N	Y	Y
D_{10}	Y	Y	Y	Y	Y	Y	A	N	Y	Y	Y	Y	Y	Y	A	Y	A	Y	N	Y	A	Y	Y

Abstain: A; No: N; Refusal: R; Yes: Y.

Phase 3. *Picture fuzzy MARCOS method for railway infrastructure ranking.*

Step 3.1. Anti-ideal and ideal railway infrastructures are determined by employing Equations (22) and (23), respectively. The picture fuzzy evaluation matrix (Table 6) is extended by adding the anti-ideal and ideal railway infrastructures. The resulting matrix is given in Table 10.

Step 3.2. The picture fuzzy additive relative importance of the anti-ideal and ideal railway infrastructures, as well as six real-life railway infrastructures, are calculated by applying the picture fuzzy Hamacher weighted average operator defined in Equation (24). The obtained values can be found in Table 11. In the base case scenario, it is assumed that the operational parameter ζ of the developed picture fuzzy group MCDM approach for risk assessment of railway infrastructure is 3.

Step 3.3. The utility degrees to the anti-ideal and ideal solutions of railway infrastructures are given in Table 11. They are calculated based on the picture fuzzy additive relative importance of railway infrastructures with the help of Equations (25) and (26).

Step 3.4. According to Equation (27), the utility function to the anti-ideal solution is 0.708. On the other hand, the utility function to the ideal solution is 0.292. This value is obtained by using Equation (28).

Step 3.5. Table 11 provides the utility function for the assessed railway infrastructures. The values are calculated with the help of Equation (29).

Step 3.6. Six railway infrastructures in the Czech Republic context are ranked according to the increasing values of the utility functions (Table 11). The ranking order is A_6 (Jaromer) $\succ$ A_2 (Hradec Kralove) $\succ$ A_5 (Chomutov) $\succ$ A_3 (Kolin) $\succ$ A_1 (Pardubice) $\succ$ A_4 (Usti nad Labem). As a result, the presented picture fuzzy

group MCDM approach for risk assessment of railway infrastructure ranked "Jaromer" (A_6) as the safest. Additionally, it is found that "Usti nad Labem" (A_4) has the highest risk in the Czech Republic context.

Table 8. The picture fuzzy risk factor importance matrix.

Risk Factor	Degree of Positive Membership	Degree of Neutral Membership	Degree of Negative Membership	Degree of Refusal Membership
C_1	0.1	0	0.6	0.3
C_2	0.8	0.2	0	0
C_3	1	0	0	0
C_4	0.5	0.4	0.1	0
C_5	0.8	0.2	0	0
C_6	0.9	0.1	0	0
C_7	0	0.1	0.8	0.1
C_8	0	0	0.9	0.1
C_9	0.2	0.4	0.3	0.1
C_{10}	0.7	0.3	0	0
C_{11}	0.8	0	0.2	0
C_{12}	0.6	0.4	0	0
C_{13}	0.5	0.4	0.1	0
C_{14}	0.1	0.4	0.5	0
C_{15}	0	0.4	0.6	0
C_{16}	1	0	0	0
C_{17}	0.7	0.3	0	0
C_{18}	1	0	0	0
C_{19}	0.2	0.6	0.2	0
C_{20}	0.9	0.1	0	0
C_{21}	0.2	0.6	0.2	0
C_{22}	1	0	0	0
C_{23}	0.9	0.1	0	0

Table 9. Subjective, objective, and hybrid risk factor importance.

Risk Factor	Subjective Importance		Objective Importance		Hybrid Importance	
	Value	Rank	Value	Rank	Value	Rank
C_1	0.0113	11	0.0573	5	0.0343	20
C_2	0.0579	3	0.0161	22	0.0370	19
C_3	0.0644	1	0.0463	11	0.0554	1
C_4	0.0451	6	0.0466	10	0.0459	12
C_5	0.0579	3	0.0270	20	0.0425	15
C_6	0.0612	2	0.0308	18	0.0460	11
C_7	0.0039	12	0.0480	9	0.0260	23
C_8	0.0003	13	0.0560	6	0.0282	22
C_9	0.0286	8	0.0526	7	0.0406	16
C_{10}	0.0547	4	0.0202	21	0.0375	18
C_{11}	0.0515	5	0.0409	13	0.0462	9
C_{12}	0.0515	5	0.0515	8	0.0515	4
C_{13}	0.0451	6	0.0317	17	0.0384	17
C_{14}	0.0193	9	0.0767	1	0.0480	8
C_{15}	0.0129	10	0.0746	2	0.0438	14
C_{16}	0.0644	1	0.0382	14	0.0513	5
C_{17}	0.0547	4	0.0120	23	0.0334	21
C_{18}	0.0644	1	0.0379	15	0.0512	6
C_{19}	0.0322	7	0.0600	4	0.0461	10
C_{20}	0.0612	2	0.0349	16	0.0481	7
C_{21}	0.0322	7	0.0715	3	0.0519	2
C_{22}	0.0644	1	0.0272	19	0.0458	13
C_{23}	0.0612	2	0.0424	12	0.0518	2

Table 10. The picture fuzzy extended evaluation matrix.

Risk Factor	Railway Infrastructure						
	A_0 (*Anti-Ideal*)	A_1	A_2	A_3	⋯	A_6	A_7 (*Ideal*)
C_1	<0, 0.1, 0.7>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	⋯	<0, 0.1, 0.7>	<0, 0.1, 0.8>
C_2	<0.5, 0.1, 0.1>	<0.5, 0.4, 0.1>	<0.3, 0.3, 0.4>	<0.5, 0.2, 0.3>	⋯	<0.4, 0.1, 0.5>	<0.3, 0.1, 0.5>
C_3	<0.5, 0.3, 0.1>	<0.1, 0.8, 0.1>	<0.2, 0.6, 0.2>	<0.1, 0.8, 0.1>	⋯	<0, 0.3, 0.7>	<0, 0.3, 0.7>
C_4	<0.4, 0.2, 0.1>	<0, 0.8, 0.2>	<0.1, 0.5, 0.4>	<0, 0.4, 0.6>	⋯	<0, 0.8, 0.2>	<0, 0.2, 0.7>
C_5	<0.6, 0.2, 0.1>	<0.2, 0.6, 0.2>	<0.4, 0.2, 0.4>	<0.3, 0.4, 0.3>	⋯	<0.4, 0.5, 0.1>	<0.2, 0.2, 0.4>
C_6	<0.5, 0.4, 0.1>	<0.4, 0.5, 0.1>	<0.3, 0.6, 0.1>	<0.4, 0.5, 0.1>	⋯	<0.2, 0.7, 0.1>	<0.2, 0.4, 0.1>
C_7	<0.1, 0.1, 0.7>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	<0, 0.1, 0.8>	⋯	<0.1, 0.1, 0.7>	<0, 0.1, 0.8>
C_8	<0, 0.1, 0.7>	<0, 0.2, 0.7>	<0, 0.1, 0.8>	<0, 0.1, 0.9>	⋯	<0, 0.2, 0.7>	<0, 0.1, 0.9>
C_9	<0.1, 0.1, 0.6>	<0, 0.2, 0.7>	<0, 0.4, 0.6>	<0, 0.3, 0.7>	⋯	<0, 0.2, 0.8>	<0, 0.1, 0.8>
C_{10}	<0.6, 0.2, 0.1>	<0.6, 0.3, 0.1>	<0.3, 0.3, 0.4>	<0.3, 0.4, 0.3>	⋯	<0.4, 0.2, 0.4>	<0.3, 0.2, 0.4>
C_{11}	<0.8, 0.1, 0.1>	<0.7, 0.2, 0.1>	<0.5, 0.4, 0.1>	<0.8, 0.1, 0.1>	⋯	<0.6, 0.2, 0.2>	<0.5, 0.1, 0.2>
C_{12}	<0.2, 0.1, 0.3>	<0, 0.1, 0.9>	<0.1, 0.3, 0.6>	<0, 0.2, 0.8>	⋯	<0, 0.4, 0.6>	<0, 0.1, 0.9>
C_{13}	<0.6, 0.3, 0.1>	<0.5, 0.4, 0.1>	<0.1, 0.3, 0.6>	<0.6, 0.3, 0.1>	⋯	<0.1, 0.4, 0.5>	<0.1, 0.3, 0.6>
C_{14}	<0, 0.1, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.1, 0.9>	⋯	<0, 0.1, 0.9>	<0, 0.1, 0.9>
C_{15}	<0, 0.1, 0.7>	<0, 0.2, 0.8>	<0, 0.1, 0.9>	<0, 0.2, 0.8>	⋯	<0, 0.3, 0.7>	<0, 0.1, 0.9>
C_{16}	<0.8, 0.1, 0.1>	<0.8, 0.1, 0.1>	<0.6, 0.1, 0.3>	<0.6, 0.2, 0.2>	⋯	<0.1, 0.3, 0.6>	<0.1, 0.1, 0.6>
C_{17}	<0.2, 0.1, 0.4>	<0.2, 0.3, 0.4>	<0.2, 0.3, 0.4>	<0.1, 0.4, 0.4>	⋯	<0.1, 0.1, 0.7>	<0, 0.1, 0.7>
C_{18}	<0.7, 0.1, 0.1>	<0.5, 0.4, 0.1>	<0.7, 0.2, 0.1>	<0.5, 0.4, 0.1>	⋯	<0.7, 0.1, 0.2>	<0.5, 0.1, 0.2>
C_{19}	<0.1, 0.1, 0.7>	<0, 0.2, 0.8>	<0, 0.3, 0.7>	<0.1, 0.1, 0.8>	⋯	<0, 0.2, 0.8>	<0, 0.1, 0.8>
C_{20}	<0.3, 0.2, 0.1>	<0.3, 0.6, 0.1>	<0.1, 0.5, 0.4>	<0.3, 0.6, 0.1>	⋯	<0.1, 0.2, 0.7>	<0.1, 0.2, 0.7>
C_{21}	<0.1, 0.1, 0.7>	<0, 0.1, 0.8>	<0, 0.2, 0.8>	<0, 0.1, 0.9>	⋯	<0, 0.1, 0.9>	<0, 0.1, 0.9>
C_{22}	<0.6, 0.1, 0.3>	<0.6, 0.1, 0.3>	<0.2, 0.5, 0.3>	<0.6, 0.1, 0.3>	⋯	<0.3, 0.1, 0.6>	<0.2, 0.1, 0.6>
C_{23}	<0.8, 0.1, 0.1>	<0.7, 0.2, 0.1>	<0.7, 0.1, 0.2>	<0.6, 0.3, 0.1>	⋯	<0.7, 0.2, 0.1>	<0.6, 0.1, 0.2>

Table 11. The railway infrastructure additive relative importance, utility degrees, utility functions, and ranks.

Railway Infrastructure	Picture Fuzzy Additive Relative Importance	Utility Degree to AIS	Utility Degree to IS	Utility Function	Rank
A_0 (anti-ideal)	<0.417, 0.135, 0.239>	1.0	2.421	-	-
A_1	<0.291, 0.265, 0.287>	0.827	2.001	0.737	5
A_2	<0.23, 0.263, 0.41>	0.660	1.599	0.589	2
A_3	<0.273, 0.243, 0.35>	0.751	1.818	0.670	4
A_4	<0.334, 0.25, 0.268>	0.885	2.143	0.789	6
A_5	<0.275, 0.199, 0.412>	0.697	1.688	0.622	3
A_6	<0.193, 0.222, 0.474>	0.566	1.370	0.505	1
A_7 (ideal)	<0.141, 0.135, 0.574>	0.413	1.0	-	-

Anti-ideal solution: AIS; Ideal solution: IS.

6.2. Sensitivity Analyses

The sensitivity analyses to changes in the trade-off and operational parameters of the presented picture fuzzy group MCDM approach are performed to check the robustness of the obtained railway infrastructure risk assessment results.

The first sensitivity analysis thoroughly explores how the trade-off parameter γ of the novel picture fuzzy hybrid method influences the ranking order of the railway infrastructures in the Czech Republic context. This method based on the direct rating and Tsallis–Havrda–Charvát entropy prioritizes the risk factors influencing railway infrastructure. More detailed, the subjective importance of a risk factor is based on importance evaluations of that risk factor by the invited railway planners. Its value is computed by using the picture fuzzy direct rating method. On the other hand, the objective importance of a risk factor is based on the railway planners' voting on six railway infrastructures. Its value is calculated by using the picture fuzzy Tsallis–Havrda–Charvát entropy method. Since different input data are used to obtain the subjective and objective ranking orders as well as the number of considered risk factors is large, significant differences in the ranking order of the risk factors are expected.

The parameter γ allows practitioners to make a trade-off between the picture fuzzy direct rating and Tsallis–Havrda–Charvát entropy methods; i.e., a compromise amongst subjective and objective importance of the risk factors. In this regard, the parameter γ is changed in the range [0, 1] with an

increment value of 0.1 (Figure 4). When $\gamma = 1$, only the picture fuzzy direct rating method is applied to subjectively prioritize the risk factors. When $\gamma = 0$, the picture fuzzy Tsallis–Havrda–Charvát entropy method is solely used to objectively evaluate the risk factors. Therefore, in the base case scenario, γ was set to 0.5 to equally appraise both methods and generate hybrid risk factor importance. According to Figure 4, "Jaromer" (A_6) is the best railway infrastructure under all γ values, since its utility function has the lowest value. Moreover, there is no change in the ranks of any railway infrastructure in all 10 new test cases; i.e., the railway infrastructure ranking order is $A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$. As a result, it is identified that the railway infrastructure risk assessment results of the investigated real-life context are very stable to changes in the trade-off parameter γ.

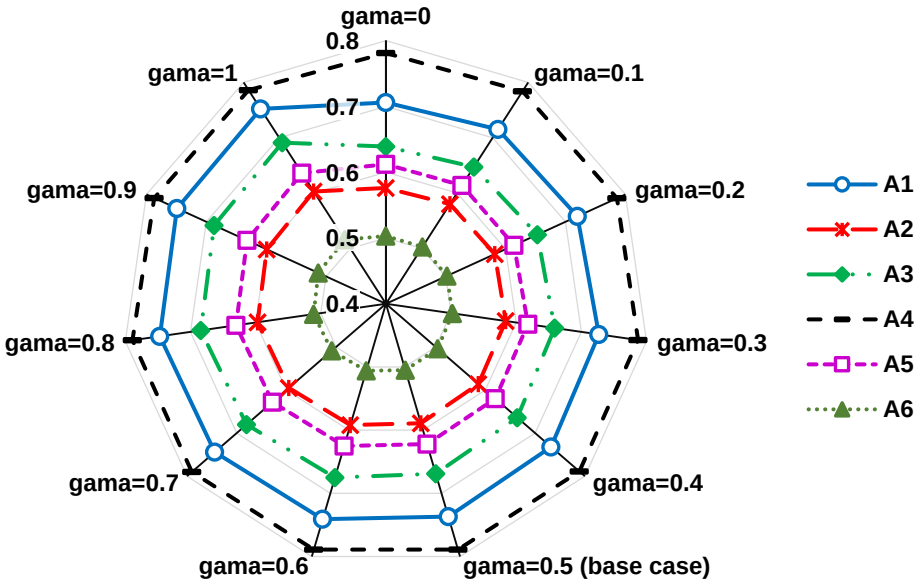

Figure 4. The sensitivity analysis to changes in the trade-off parameter γ.

The second sensitivity analysis is performed to check the robustness of generated solutions to changes in the operational parameter ζ of the developed picture fuzzy group MCDM approach for risk assessment of railway infrastructure (Figure 5). When $\zeta = 1$, the Hamacher T-norm and T-conorm reduce to probabilistic T-norm and T-conorm, respectively. When $\zeta = 2$, the Hamacher T-norm and T-conorm reduce to Einstein T-norm and T-conorm, respectively. As a result, in the base case scenario, it was assumed that ζ is 3. Ten additional test cases are created to systematically analyze the influence of the operational parameter; i.e., $\zeta \in \{0.5, 1, 1.5, 2, 2.5, 3.5, 4, 4.5, 5, 5.5\}$. In all additional test cases, the ranking order of the railway infrastructures in the Czech Republic context is $A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$ (Figure 5). This result was obtained in the base case scenario. It is found that the ranks of the railway infrastructures are very stable to changes in the operational parameter ζ.

According to the results of both sensitivity analyses, it can be outlined that the formulated picture fuzzy group MCDM approach for risk assessment of railway infrastructure is highly robust.

6.3. Comparative Analysis

The comparative analysis is performed to check the reliability of the presented results of the developed picture fuzzy group MCDM approach for risk assessment of railway infrastructure. The railway infrastructure risk assessment problem in the Czech Republic context is solved with five available state-of-the-art picture fuzzy approaches: (1) picture fuzzy TOPSIS [76], (2) picture fuzzy EDAS [77], (3) picture fuzzy grey relational analysis [78], (4) picture fuzzy grey relational projection [79], and (5) picture fuzzy cross-entropy [80].

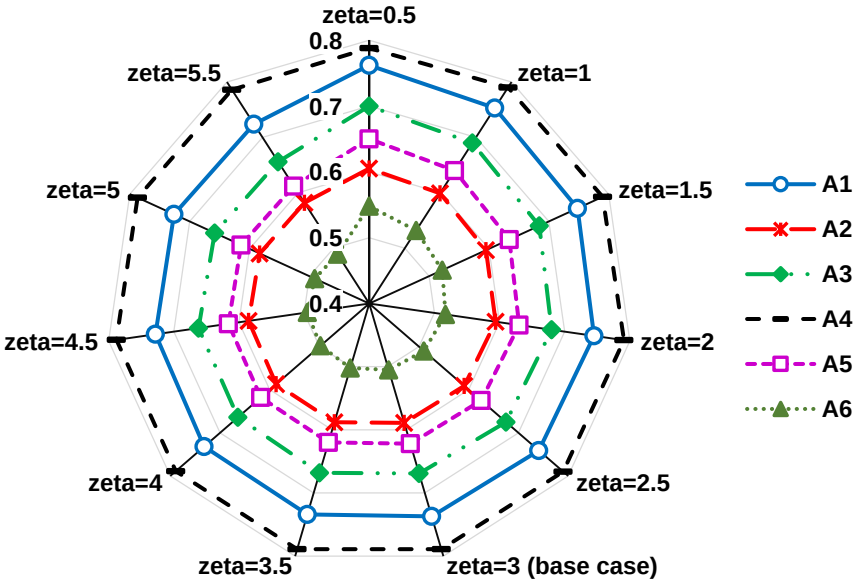

Figure 5. The sensitivity analysis to changes in the operational parameter ζ.

The comparison results are presented in Table 12. The developed picture fuzzy group MCDM approach and picture fuzzy grey relational projection generate the identical ranking order of the railway infrastructures; i.e., A_6 (Jaromer) $\succ$ A_2 (Hradec Kralove) $\succ$ A_5 (Chomutov) $\succ$ A_3 (Kolin) $\succ$ A_1 (Pardubice) $\succ$ A_4 (Usti nad Labem). On the other hand, the picture fuzzy TOPSIS, grey relational analysis, and cross-entropy produce the same ranking order; i.e., A_6 (Jaromer) $\succ$ A_5 (Chomutov) $\succ$ A_2 (Hradec Kralove) $\succ$ A_3 (Kolin) $\succ$ A_1 (Pardubice) $\succ$ A_4 (Usti nad Labem). Six compared approaches assess "Jaromer" (A_6) as the safest railway infrastructure of the analyzed real-life context. The proposed approach, picture fuzzy grey relational projection and picture fuzzy EDAS, rank "Hradec Kralove" (A_2) as the second-best railway infrastructure. The other three approaches put this railway infrastructure in the third position and evaluate "Chomutov" (A_5) as the second-best. "Kolin" (A_3) holds the fourth position in five out of six approaches. Only the picture fuzzy EDAS puts this railway infrastructure in the third place. "Pardubice" (A_1) is the second-worst railway infrastructure in five out of six approaches. Finally, "Usti nad Labem" (A_4) is the worst-ranked in all approaches.

Table 12. The comparative analysis of the state-of-the-art picture fuzzy approaches.

Approach	Ordering
Picture fuzzy group MCDM approach for risk assessment of railway infrastructure (our approach)	$A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$
Picture fuzzy TOPSIS [76]	$A_6 \succ A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$
Picture fuzzy EDAS [77]	$A_6 \succ A_2 \succ A_3 \succ A_1 \succ A_5 \succ A_4$
Picture fuzzy grey relational analysis [78]	$A_6 \succ A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$
Picture fuzzy grey relational projection [79]	$A_6 \succ A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$
Picture fuzzy cross-entropy [80]	$A_6 \succ A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$

According to the results of the comparative analysis, it can be outlined that the developed approach for risk assessment of railway infrastructure is highly reliable.

The ranking similarity between six compared approaches is examined by Spearman's rank correlation coefficient. This quantitative metric reveals the strength of the relationship between compared approaches (Table 13).

The picture fuzzy group MCDM approach for risk assessment of railway infrastructure has 93.2% of ranks matched. Additionally, there is a perfect relationship between the formulated approach and

picture fuzzy grey relational projection. Finally, a very strong correlation exists between the introduced approach and picture fuzzy TOPSIS, EDAS, grey relational analysis, and cross-entropy.

According to the results of Spearman's rank correlation analysis (Table 13), it can be outlined that the introduced approach for risk assessment of railway infrastructure produces highly consistent results.

Table 13. The ranking similarity of the compared picture fuzzy approaches.

Approach	Our Approach	PF TOPSIS	PF EDAS	PF GRA	PF GRP	PF CE	Overall
Our approach	–	*0.943*	*0.829*	*0.943*	*1*	*0.943*	*0.932*
PF TOPSIS	0.943	–	0.657	1	0.943	1	0.909
PF EDAS	0.829	0.657	–	0.657	0.829	0.657	0.726
PF GRA	0.943	1	0.657	–	0.943	1	0.909
PF GRP	1	0.943	0.829	0.943	–	0.943	0.932
PF CE	0.943	1	0.657	1	0.943	–	0.909

Cross-entropy: CE; Grey relational analysis: GRA; Grey relational projection: GRP; Picture fuzzy: PF.

7. Conclusions

The picture fuzzy group MCDM approach for risk assessment of railway infrastructure is introduced in this paper. Its major contributions are: (i) For the first time, PFSs are employed for representing railway planners' preferences and handling risk-related information; (ii) the novel picture fuzzy hybrid method based on the direct rating and Tsallis–Havrda–Charvát entropy is provided to prioritize risk factors influencing railway infrastructure; (iii) the new picture fuzzy MARCOS method is developed to rank railway infrastructures; and (iv) the formulated approach is implemented in the Czech Republic context.

The merits of the presented real-life case study are: (a) The utility of the introduced approach is demonstrated; (b) the high robustness of the formulated approach is verified by two sensitivity analyses, since the ranks of the railway infrastructures are very stable to changes in the trade-off and operational parameters; (c) the high reliability of the developed approach is approved by the comparative analysis with the picture fuzzy TOPSIS, EDAS, grey relational analysis, grey relational projection, and cross-entropy; and (d) the high consistency with five state-of-the-art picture fuzzy approaches is confirmed by Spearman's rank correlation analysis, since 93.2% of railway infrastructure ranks are matched.

The picture fuzzy group MCDM approach for risk assessment of railway infrastructure generates the following ranking order: A_6 (Jaromer) $\succ$ A_2 (Hradec Kralove) $\succ$ A_5 (Chomutov) $\succ$ A_3 (Kolin) $\succ$ A_1 (Pardubice) $\succ$ A_4 (Usti nad Labem). This novel approach identified "Jaromer" as the safest. On the other hand, the worst-ranked railway infrastructure is "Usti nad Labem". It has the highest risk in the Czech Republic context. As a result, it is strongly recommended to undertake a safety improvement project for this railway infrastructure.

The picture fuzzy grey relational analysis and grey relational projection are one-parametric approaches with build-in distinguishing coefficients. The picture fuzzy TOPSIS, EDAS, and cross -entropy have no parameters. On the other hand, the formulated approach involves three intrinsic parameters, which is highly desirable for solving the complex railway infrastructure risk assessment problem. They are the trade-off parameter γ, the operational parameter ζ, and the information measure parameter χ. As a result, compared to the state-of-the-art picture fuzzy approaches, the provided three-parametric approach has superior flexibility in assessing the risk of railway infrastructure.

Limitations of this paper can indicate its possible extension areas. The limitations are: (1) The risk factors influencing railway infrastructure are not filtered; (2) interrelationships between the risk factors are mainly ignored. A well-known technique for filtering influential factors is the Delphi method. It is traditionally used to obtain a consistent flow of answers through the results of questionnaires. Its major features are anonymous response, iteration and controlled feedback, and finally statistical group response. In a future study, an online multi-round questionnaire approach, in line with the Delphi method, could be performed to collect risk-related information and overcome the first limitation. On the

other hand, the DEMATEL method is mostly applied for modeling interrelationships of influential factors. This method has not been extended before into the picture fuzzy environment. One of the future researches may integrate a picture fuzzy DEMATEL method into the introduced methodological framework for risk assessment to handle the second limitation. Additionally, a comparison of the risk factors influencing railway infrastructure in the Czech Republic with risk factors proposed by other researchers can also be seen as an interesting topic that deserves a future research effort. Finally, the introduced methodological approach can be used not only for risk assessment of railway infrastructure but also in macro-level issues. Indicatively, various risks exist in different frames, due to climate change, like fires, floods, droughts, hurricanes, and tornadoes, which affect social and economic life seriously.

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Appendix A

Sample questionnaire—Importance of risk factors influencing railway infrastructure in the Czech Republic.

Response in each row is not required since a participant can refuse to vote on a particular risk factor. There are three responses/columns to choose from: YES—risk factor is important for railway infrastructure; ABSTAIN—if you are not certain about the importance of a risk factor; NO—risk factor is not important for railway infrastructure.

	YES	ABSTAIN	NO
(1) Prolonged drought	○	○	○
(2) Extreme temperatures	○	○	○
(3) Flood	○	○	○
(4) Heavy rainfall	○	○	○
(5) Extreme wind	○	○	○
(6) Epidemics	○	○	○
(7) Epiphytia	○	○	○
(8) Epizootics	○	○	○
(9) Disrupted food supplies	○	○	○
(10) Disrupted communication systems	○	○	○
(11) Critical information infrastructure violation	○	○	○
(12) Special flood	○	○	○
(13) Hazardous chemical leakage	○	○	○
(14) Disrupted water supply	○	○	○
(15) Disrupted gas supply	○	○	○
(16) Disrupted oil supplies	○	○	○
(17) Radiation accidents	○	○	○
(18) Disrupted electricity supply	○	○	○
(19) Migration wave	○	○	○
(20) Legality violation	○	○	○
(21) Economic destabilization	○	○	○
(22) Military assault	○	○	○
(23) Human factor failure	○	○	○

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