Design a combination of filters to reduce decline the attenuation in magnitude response

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Abstract: - Some biquad filters structures are characterized by a decreasing of the attenuation at high frequencies, caused by the final value of the output resistance of the operational amplifier. This decreasing of the attenuation occurs only for the some even orders filters, i.e. for the biquads, of which a higher order filter is composed. One possible solution is to replace the biquad with another biquad that does not have this property. The conversion of parameters of such biquads to each other biquad is described in the detailed steps. The results of the proposed numerical procedure are verified by simulation.

Key-Words: **-** Biquad, low path filter, structure SK, structure H, higher order filter, magnitude response.

1 Introduction

The low pass active RC (ARC) filters of some structures, for example Sallen-Key structure [1], [2], [3], [4], and more others, based on a polynomial from biquads supplemented for the odd-orders at the beginning with one RC circuit. Its frequency response is shown in Fig.1above, for the even-order decline the attenuation at high frequencies, as shown in Fig.1below. This effect and/or the same problem exist generally for band-pass filter Huelsmann structure and/or Deliyanis structure and for the stop-band filter, too.

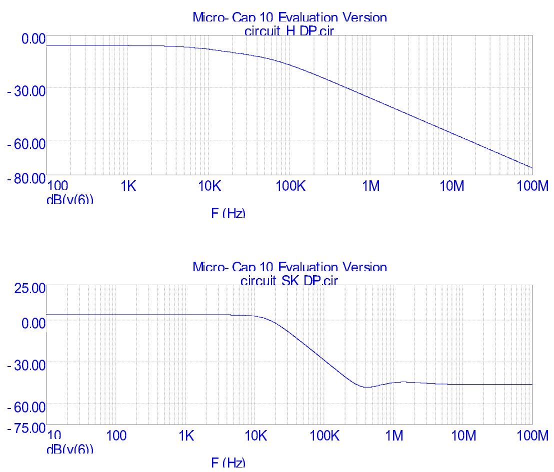

Fig.1Magnitude frequency response of Sallen and Key filter odd-orders (3st) above and even orders (2nd) below.

The cause is depicted in Fig.2 for low-pass filter. At the highest frequencies, all capacitors behave like a short circuit and the operational amplifier loses the open loop gain *A* [5].

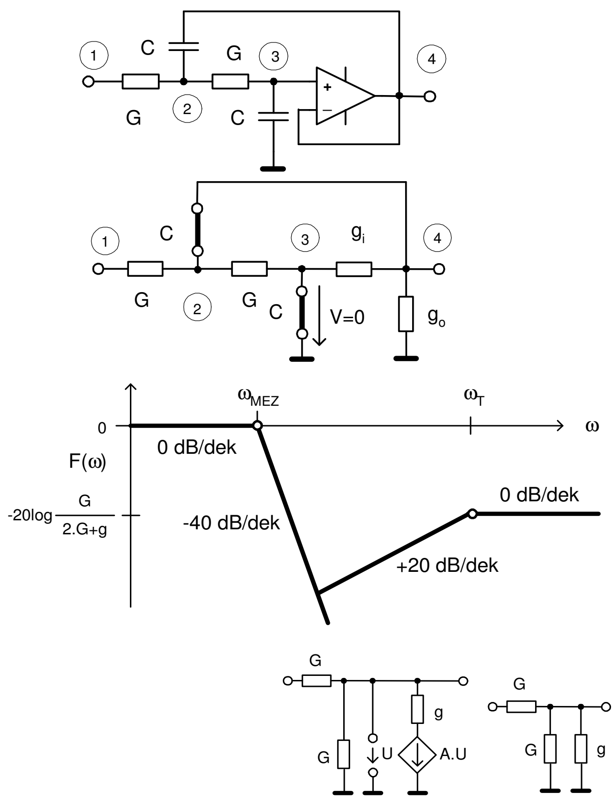

Fig.2 Biquad low-pass of the SK structure above, its model in high frequencies in the middle and its frequency response below.

The reason is a capacitor, connecting for a high-frequency, which already amplification factor of operational amplifier A is reduced to zero, input and output, as is shown in equivalent circuit in Fig.2 in the middle. Therefore, the magnitude of the voltage transfer ratio at the highest frequencies must be not equal to zero in this case (1):

$$F_{inf} = \lim_{\omega \to \infty} \frac{V_{OUT}}{V_{IN}} = \frac{-(-G)}{2 \cdot G + g} = \frac{G}{2 \cdot G + g} \neq 0 \qquad (1)$$

where: F_{inf} is the magnitude of the voltage transfer ratio close to infinity frequency, V_{IN}, V_{OUT} are the input and output voltages, g is the series output conductance of the operational amplifier, G is the conductance of a working resistor in the filter.

During the synthesis of a cascade filter of biquads one choose no as LP-SK type, but as the type LP-H (Fig.3) which does not have this described disadvantage.

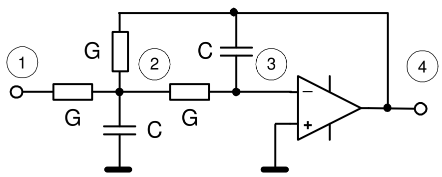

Fig.3 Biquad LP-H

The reason is grounded capacitor into input filter LP-H, as is shown in equivalent circuit for higher frequencies in stop band in Fig.4. In this case, the part of this circuit after node 2 is excited by a zero voltage, i.e. V = 0 in node 2, as is shown in Fig.4. Thus V_4 must be equal to zero, too.

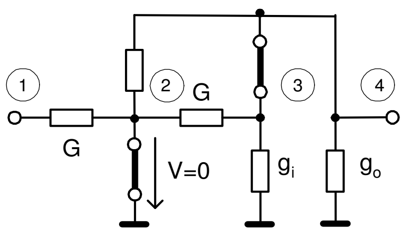

Fig.4 Model of the LP-H biquads in high frequencies

While for cascade biquads LP-SK will be reduction of attenuation in an zone over of transit frequency of the operational amplifier [3] in stop band, for cascade biquads LP-SK and LP-H then drop attenuation at high frequencies already does not occur. The magnitude characteristic has monotonous character in stop band in this case.

2 Design of the Combination Filter

The main problem with the design of such a combination filter is the recalculation of the LP-SK biquad parameters to LP-H biquad parameters.

Consider now an active RC low-pass filter with operational amplifiers and the Butterworth approximation function. The passband attenuation is A_C = 3 dB for passband frequency $f_C = 10.10^3$ Hz, for stopband frequency $f_S = 20.10^3$ Hz is stopband attenuation A_S = 30 dB. Steps of the calculation are following [6]:

1) Filter specification: is shown in Fig.5.

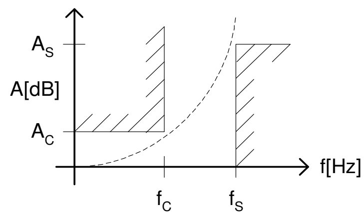

Fig.5 Filter specification

2.1 SK-filter Design

2) Coefficient of selectivity: k is given by:

$$k = \frac{f_S}{f_C} = \frac{20 \cdot 10^3}{10 \cdot 10^3} = 2$$

3) Coefficient of damping: d is given by

$$d = \frac{10^{0,1 \cdot A_S} - 1}{10^{0,1 \cdot A_C} - 1} = \frac{10^{0,1 \cdot 20} - 1}{10^{0,1 \cdot 3} - 1} = 100$$

4) Order of filter: n is given by

$$n \geq \frac{\log d}{2 \cdot \log k} = \frac{\log 100}{2 \cdot \log 2} = 3{,}32$$

$$n = 4$$

5) From table Butterworth coefficients are follows:

Table 1.Coeficients Butterworth Approximation

N	F_0	Q
4	1	0,541
	1	1,306

6) Capacity is determined by

$$C = \frac{10^{-7}}{\sqrt{f_C}} = \frac{10^{-7}}{\sqrt{10^4}} = 10^{-9}\,F$$

7) Coefficients of biquads: m are following [4]

$$m_1 = \frac{1}{Q_1^2} = \frac{1}{0{,}541^2} = 3{,}414$$

$$m_2 = \frac{1}{Q_2^2} = \frac{1}{1{,}306^2} = 0{,}585$$

8) The value of capacity for first section No.1:

$$C_1 = \frac{C}{\sqrt{m_1}} = \frac{1 \cdot 10^{-9}}{\sqrt{3{,}414}} = 541 \cdot 10^{-12}\,F$$

$$C_2 = C.\sqrt{m_1} = 1 \cdot 10^{-9} \cdot \sqrt{3{,}414} = 1{,}8.10^{-9}\,F$$

and for second one section No.2:

$$C_3 = \frac{C}{\sqrt{m_2}} = \frac{1 \cdot 10^{-9}}{\sqrt{0{,}585}} = 1{,}3 \cdot 10^{-9}\,F$$

$$C_4 = C \cdot \sqrt{m_2} = 1 \cdot 10^{-9} \cdot \sqrt{0{,}585} = 764 \cdot 10^{-12}\,F$$

9) The values for both biquads:

$$n_1 = \frac{1}{m_1 \cdot Q_1^2} = \frac{1}{3{,}414 \cdot 0{,}541^2} = 0{,}853$$

$$n_2 = \frac{1}{m_2 \cdot Q_2^2} = \frac{1}{0{,}585 \cdot 1{,}306^2} = 1$$

10) The resistance of filter resistors: based on relationship

$$R = \frac{1}{2 \cdot \pi \cdot f_C \cdot C} = \frac{1}{2 \cdot \pi \cdot 10^4 \cdot 10^{-9}} = 15 \cdot 10^3\,\Omega$$

are resistance following:

$$R_1 = \frac{R}{\sqrt{n_1}} = \frac{15 \cdot 10^3}{\sqrt{0{,}853}} = 16 \cdot 10^3\,\Omega$$

$$R_2 = R \cdot \sqrt{n_1} = 15 \cdot 10^3 \cdot \sqrt{0{,}853} = 13{,}8 \cdot 10^3\,\Omega$$

$$R_5 = \frac{R}{\sqrt{n_2}} = \frac{15 \cdot 10^3}{\sqrt{1}} = 15 \cdot 10^3\,\Omega$$

$$R_6 = R \cdot \sqrt{n_2} = 15 \cdot 10^3 \cdot \sqrt{1} = 15 \cdot 10^3\,\Omega$$

11) Coefficients for resistors of amplifiers:

$$r_1 = \frac{1}{Q_1^2} = \frac{1}{0{,}541^2} = 3{,}414$$

$$r_2 = \frac{1}{Q_2^2} = \frac{1}{1{,}306^2} = 0{,}585$$

12) The resistance of amplifier resistors:

$$R_3 = \frac{10^3}{\sqrt{r_1}} = \frac{10^3}{\sqrt{3{,}414}} = 540\Omega$$

$$R_4 = 10^3 \cdot \sqrt{r_1} = 10^3 \cdot \sqrt{3{,}414} = 1{,}8 \cdot 10^3\,\Omega$$

$$R_7 = \frac{10^3}{\sqrt{r_2}} = \frac{10^3}{\sqrt{0{,}585}} = 1{,}3 \cdot 10^3\,\Omega$$

$$R_8 = 10^3 \cdot \sqrt{r_2} = 10^3 \cdot \sqrt{0{,}585} = 765\Omega$$

13) Circuit diagram of calculated filter is shown in Fig.6.

14) Simulation of calculated results by spice-like program MC-10 is shown in Fig.7.

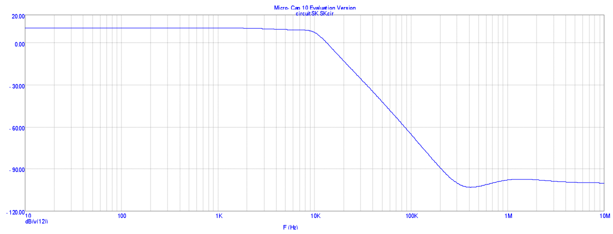

Fig.7 Simulation result of LP-SK 4th order from biquads LP-SK only.

2.2 Recalculation SK into H Filter

15) Recalculation SK parameters into H parametrs consists from folloving steps:

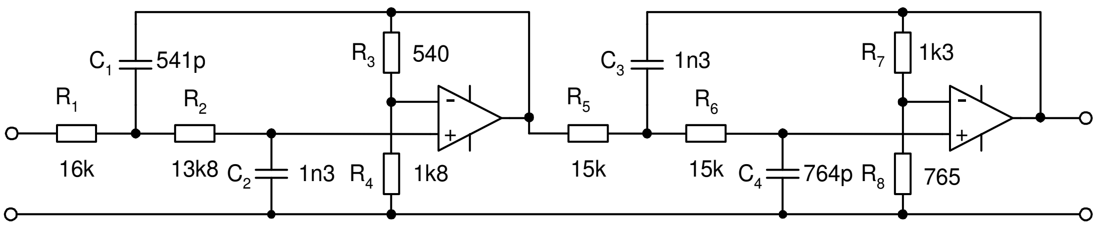

Fig.6 Calculated filter LP-SK 4th order

16) Calculation of first section amplifying [7]

$$K = 1 + \frac{R_3}{R_4} = 1 + \frac{540}{1{,}8 \cdot 10^3} = 1{,}277$$

17) Coefficient of Huelsmann biquad LP-H [4]

$$m \le \frac{1}{4 \cdot Q^2 \cdot (1 - K)} =$$

$$= \frac{1}{4 \cdot 0{,}541^2 \cdot (1 - 1{,}277)} = 3{,}081$$

18) The capacitance of capacitors of Huelsman biquad

$$C_1 = C = 1 \cdot 10^{-9}\,F$$

$$C_2 = m \cdot C = 3{,}081 \cdot 1 \cdot 10^{-9} = 3{,}081 \cdot 10^{-9}\,F$$

19) The resistance of the resistors of Huelsman biquad

$$R_2 = \frac{1 \pm \sqrt{1 - 4 \cdot (1 - K) \cdot Q^2 \cdot m}}{4 \cdot \pi \cdot f_C \cdot Q \cdot m \cdot C} =$$

$$= \frac{1 \pm \sqrt{1 - 4 \cdot (1 - 1{,}277) \cdot 0{,}541^2 \cdot 3{,}081}}{4 \cdot \pi \cdot 10 \cdot 10^3 \cdot 0{,}541 \cdot 3{,}081 \cdot 1 \cdot 10^{-9}} =$$

$$= 4{,}77 \cdot 10^3\,\Omega$$

$$R_1 = \frac{R_2}{K} = \frac{4{,}77 \cdot 10^3}{1{,}277} = 3{,}73 \cdot 10^3\,\Omega$$

$$R_3 = \frac{1}{4 \cdot \pi^2 \cdot f_C^2 \cdot R_2 \cdot m \cdot C^2} =$$

$$\frac{1}{4 \cdot \pi^2 \cdot (10 \cdot 10^3)^2 \cdot 4{,}77 \cdot 10^3 \cdot 3{,}081 \cdot (1 \cdot 10^{-9})^2} =$$

$$= 1{,}72 \cdot 10^3\,\Omega$$

20) Circuit diagram of combined filter designed above is depicted in Fig.8.

21) Results of simulation by spice-like program MC-10 is shown in following Fig.9.

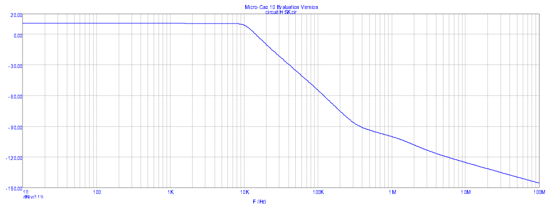

Fig.9 Simulation result of LP 4th order from biquads LP-H and LP-SK.

2.3 Nonzero Transfer Correction

22) Correction nonzero magnitude transfer for lowest frequencies [4], [8] is based on the value obtained by the simulation (step 21) in Fig.9.

Rezistor R_4 will be substituted by the voltage divider R_{41}, R_{42}. For the difference in magnitude 10 dB (see Fig.9) is:

$$10dB = 20 \cdot \log \frac{\tilde{V}}{V}$$

$$3{,}17 = \frac{\tilde{V}}{V} = \frac{R_{41} + R_{42}}{R_{42}}$$

Thus set of equations to calculate the voltage divider is following

$$\frac{R_{41} + R_{42}}{R_{42}} = 3{,}17$$

$$\frac{R_{41} \cdot R_{42}}{R_{41} + R_{42}} = R_4 = 15 \cdot 10^3$$

$$R_{41} + R_{42} = 3{,}17 \cdot R_{42}$$

$$R_{41} = R_{42} \cdot (3{,}17 - 1) = R_{42} \cdot 2{,}17$$

$$\frac{R_{42} \cdot 2{,}17 \cdot R_{42}}{R_{42} \cdot 2{,}17 + R_{42}} = \frac{R_{42} \cdot 2{,}17}{3{,}17} = 15 \cdot 10^3$$

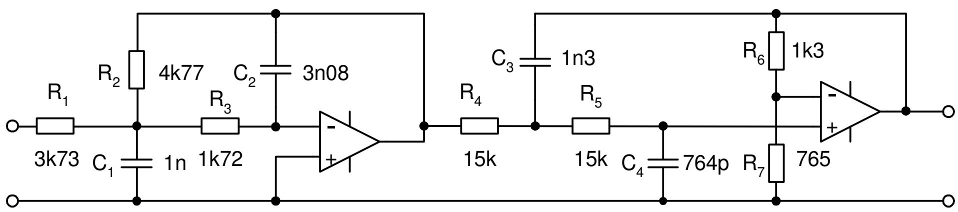

Fig.8 Recalculated filter LP 4th order consists from two different biquads LP-H and LP-SK.

$$R_{42} = \frac{3{,}17}{2{,}17} \cdot 15 \cdot 10^3 = 21{,}9 \cdot 10^3 \Omega$$

$$R_{41} = R_{42} \cdot 2{,}17 = 21{,}9 \cdot 10^3 \cdot 2{,}17 = 47{,}5 \cdot 10^3 \Omega$$

23) Final circuit diagram after correction with voltage divider is shown in Fig.10.

24) Results of simulation after correction by spice-like program MC-10 is shown in Fig.11.

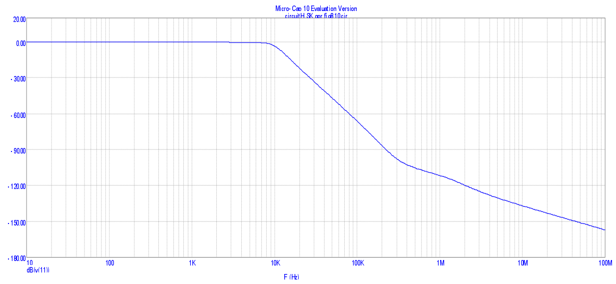

Fig.11. Simulation result of proposed finally circuit

However larger dynamics according to [4] reaches when partial biquad, sorted by increasing the quality Q. Since the filter LP-SK with voltage follower has the quality $Q < 15$ and filter LP-H has the quality $Q < 10$, it appears advantageous to include from this reason biquad LP-H at the beginning of the cascade biquads SK. But from the viewpoint of the noise i.e. high frequencies would be preferable to include biquad LP-H at the end of the cascade, so that the grounded capacitor seduced these high frequencies to ground. The noise F [9], [10], [11] is:

$$F = F_1 + \frac{F_2 - 1}{A_1} + \frac{F_3 - 1}{A_1.A_2} + \ldots + \frac{F_n - 1}{\prod_{i=1}^{n-1} A_i} \qquad (2)$$

While the magnitude response was discussed above, turn now our attention into phase response [4]. For this reason are in Fig.12 phase frequency response of filter LP 4[th] order which consists from two biquads LP-SK in cascade, in Fig. 13 and 14 phase response results of the filter LP 4[th] order which consists from biquads LP-H and LP-SK in cascade after correction nonzero transfer for lowest frequencies, respectively. As can be seen, the phase responses are after adding 180° (by inverting operation amplifier of Huelsmann biquad) identical.

3 Conclusion

As shown by the comparison of two very detailed characteristics in Fig.12 and Fig.14, the filter calculated by the proposed procedure has the same phase response, but drop attenuation at high frequencies already does not occur.

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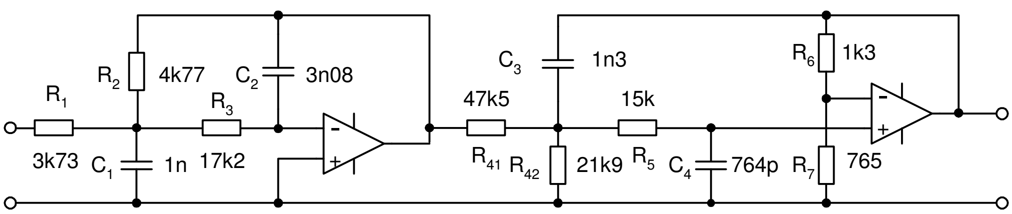

Fig.10. Recalculated circuit with voltage divider after correction nonzero transfer for lowest frequencies

Appendix

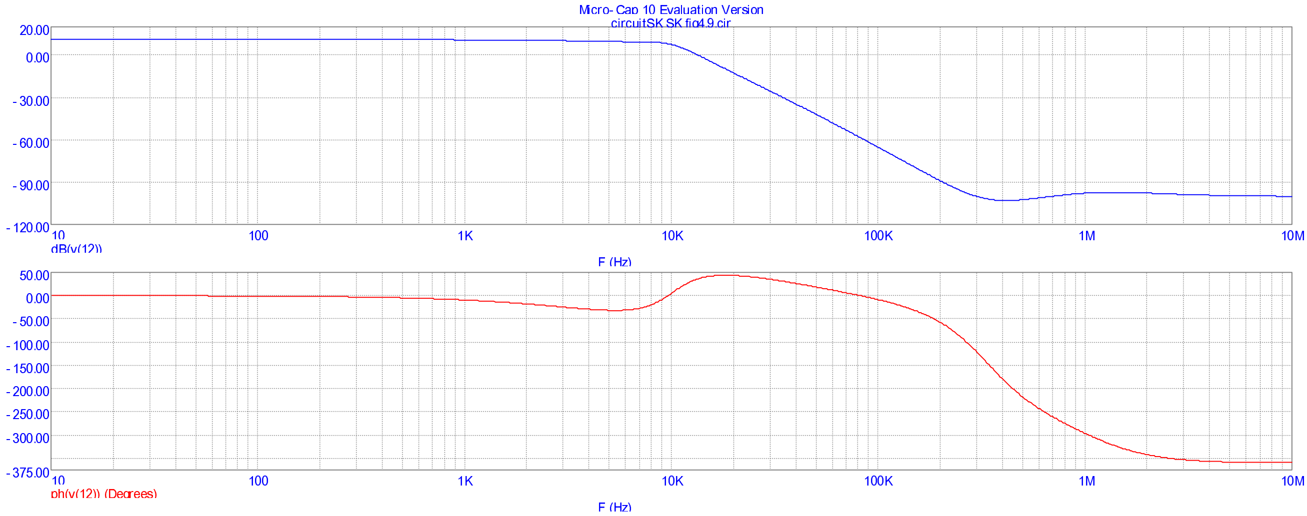

Fig.12 Magnitude and phase frequency response of filter LP 4th order which consists from two biquads LP-SK

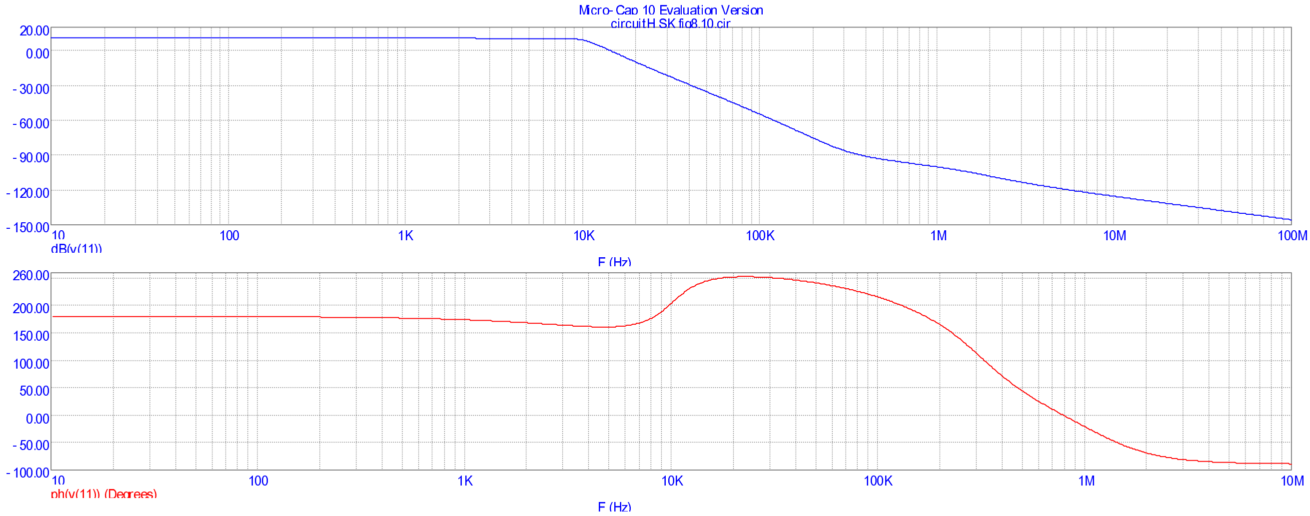

Fig.13 Magnitude and phase response of filter LP 4th order which consists from LP-H and LP-SK in cascade.

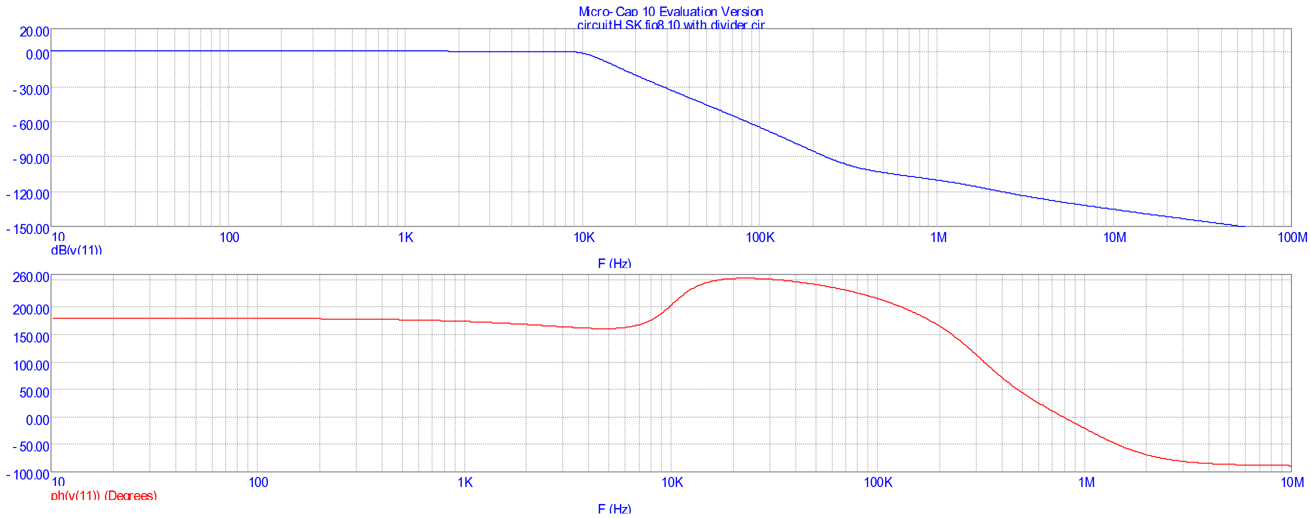

Fig.14 Magnitude and phase frequency response of filter LP 4th order which consists from biquads LP-H and LP-SK in cascade after correction nonzero transfer for lowest frequencies.