

# Practical Relationships between Propagation and Meteorological Variables Derived from Videodistrometer Measurement and Its Ambiguity Discussion — Examples from Prague (CZ)

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**Abstract**— In this contribution we discuss the possibilities of the distrometer or videodistrometer measurement. Firstly we describe briefly what outputs are given through the videodistrometer measurement: equivolumetric rain drop diameter, fall velocity, oblateness etc. From these quantities we derive the one minute drop size distributions (DSDs) and from it pertinent radar reflectivity factors  $Z$ , rain rates  $R$ , specific rain attenuations of hypothetical microwave link  $A$  and rain kinetic energies  $E_k$ . 2017 and 2018 videodistrometer data from Prague were applied. Scatterplots  $E_k R$ ,  $E_k Z$ ,  $A-Z$  and  $A-R$  are shown. In two last relations the unambiguity as a function of frequency was discussed it was observed that the unambiguity is maximal at 25 GHz frequency and minimal at 100 GHz for investigated frequencies from 10 to 100 GHz. For  $A-R$  relations the unambiguity was estimated also through the coefficient of determination for the power law approximation.

## 1. INTRODUCTION

The DSD can be measured by radar [1] but distrometer give us more info about rain. The videodistrometer (Fig. 1.) measurement enables us not only to measure the rain drop properties like rain drop diameter ( $D$ ), fall velocity ( $v$ ), oblateness ( $o$ ) or rain drop shape. Subsequently, using pertinent equations, from these quantities we can derive the drop size distribution ( $DSD$ ), rain rate ( $R$ ), radar reflectivity factor ( $Z$ ), kinetic energy [2] of rain ( $E_k$ ), specific rain attenuation of (hypothetical) microwave link ( $A$ ) and others:

$$R_g = \frac{3.6}{10^3} \cdot \pi \cdot \int_0^{\infty} D^3 \cdot v(D) \cdot N(D) \cdot dD, \quad (1)$$

where  $R_g$  is the rain rate (corresponding to the rain gauge rain rate),  
 $v(D)$  is the terminal rain drop falling velocity,  
 $N(D)$  is the drop size distribution,  
 $D$  is the equivolumetric rain drop radius.



Figure 1. Videodistrometer made in Joaneum Research Graz, Austria.

For the specific rain attenuation  $A$  in [dB/km] next equation is used:

$$A = 8.686 \cdot 10^3 \cdot \lambda \cdot Im \int f(D) \cdot N(D) \cdot dD \quad [\text{dB/km}], \quad (2)$$

where  $f$  is the complex forward scattering function,  $\lambda$  is the wave length,  $Im$  means the imaginary part.

The radar reflectivity factor  $z$  is defined by next equation:

$$z = \int_0^{\infty} D^6 \cdot N(D) \cdot dD \quad [\text{mm}^6 \text{m}^{-3}], \quad (3)$$

For the radar reflectivity factor is logarithmic unit  $Z$  (capital letter “Z”) [dBz] usually used:

$$Z = 10 \cdot \log_{10} z = 10 \cdot \log_{10} \left\{ \int_0^{\infty} D^6 \cdot N(D) \cdot dD \right\} \quad [\text{dBz}], \quad (4)$$

The rain kinetic energy (expressed in Joules/h per meter squared) of a raindrop is defined as:

$$E_k = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{12} \cdot 10^{-3} \cdot \pi \cdot \rho \cdot \int_0^{\infty} D^3 \cdot v(D)^2 \cdot N_2(D) \cdot dD \quad (5)$$

where  $m$  is mass [g],

$\rho$  is the density of water [ $\text{g} \cdot \text{cm}^{-3}$ ],

$v$  is velocity [ $\text{m} \cdot \text{s}^{-1}$ ],

$D$  is diameter [mm],

$N_2(D)$  is 2D drop size distribution.

We are focusing preferably on dependence of specific rain attenuation on rain rate ( $A$ - $R$ ) and on unambiguity of this dependence for various frequencies. We found frequencies where the  $A$ - $R$  dependence is unambiguous. We are trying to make these approximations more complex but simple as it is possible. The main purpose of estimated equations is the simple practical calculation of specific rain attenuation for defined frequencies. For more about DSD see [3–8].

There are, in fact, two different types of DSD:

1. “2D DSD”  $N_2(D)$  — this DSD corresponds to the number of raindrops of certain size per unit plane. This is directly measured by distrometer. The rain drop fall velocity is not considered. This DSD plays role for the rain kinetic energy  $E_k$  or rain rate  $R$  computation.
2. “3D DSD”  $N(D)$  informing us about number of raindrops of certain size per unit volume. The rain drop fall velocity is determining the vertical distance between drops plays important role. This 3D DSD must be computed from 2D DSD (divide the 2D DSD by corresponding rain drop fall velocity — we prefer the computed fall velocity depending on drop size to the measured fall velocity). The 3D DSD is required for radar reflectivity factor or specific rain attenuation (of radio signal) computation.

## 2. RELATIONSHIPS OF NO FREQUENCY DEPENDENCE

All relationships derived in this part as well as well in Chapter 3 correspond to the videodistrometer measurements in Prague in 2017 and 2018. All derived values correspond to the mean one minute DSD (drop Size Distribution).

### 2.1. Relationship Kinetic Energy of Rain $E_k$ — Radar Reflectivity Factor $Z$

Scatterplot of this dependence is in Fig. 2 As one can see, this  $E_k$  —  $Z$  dependence is ambiguous.

### 2.2. Relationship Kinetic Energy of Rain $E_k$ — Rain Rate $R$

This relationship is a little bit unambiguous, see scatterplot in Fig. 3.

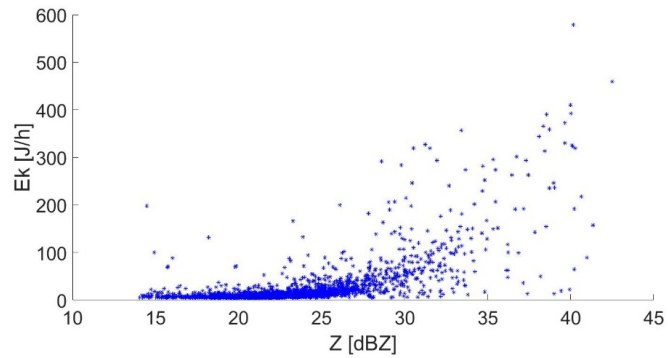


Figure 2. Scatterplot of relationship Kinetic energy of rain  $E_k$  — Radar reflectivity factor  $Z$  ( $E_k$ - $Z$ ).

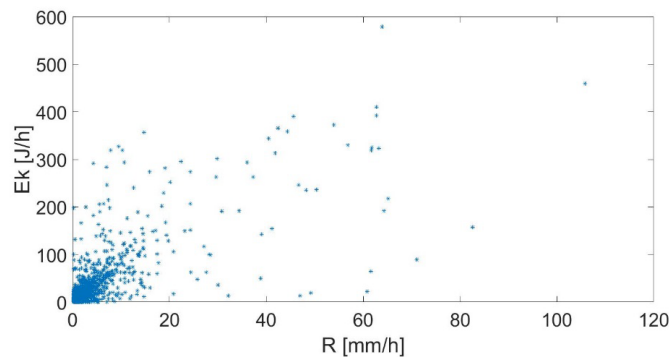


Figure 3. Scatterplot of relationship Kinetic energy of rain  $E_k$  — Rain rate  $R$  ( $E_k$ - $R$ ).

### 3. RELATIONSHIPS DEPENDENT ON FREQUENCY

#### 3.1. Specific Rain Attenuation $A$ – Rain Rate $R$ Dependence ( $A$ - $R$ )

Having computed the specific rain attenuation for required frequencies (equation 2) and rain rate (equation 1) we prepared a set of scatterplots for frequencies 10, 15, 20, 25, ... 95 and 100 GHz.

After finishing  $A$ - $R$  relation we concluded that the power law approximation underestimates the actual values (Fig. 4). Therefore we started to consider regression from the rain rate values only above 10 mm/h which are also more important from the rain attenuation point of view (Fig. 5). One can see that the approximation which raised from rain rate values above (or equal) 10 mm/h fits better the measured points. Therefore we prepared all power-law regression curves considering rain rates above 10 mm/h (Figs. 6–11).

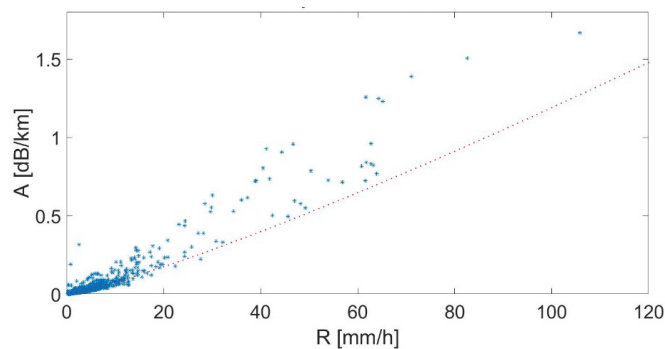


Figure 4.  $A$ - $R$  scatterplot for 10 GHz. Red dotted curve is the power-law approximation.

The unambiguity is increasing with frequency from 10 to 25 GHz and decreasing with frequency 30–100 GHz. The unambiguity is the best one for frequency 25 GHz.

For the objective assessment of the unambiguity we will use two testing parameters. The

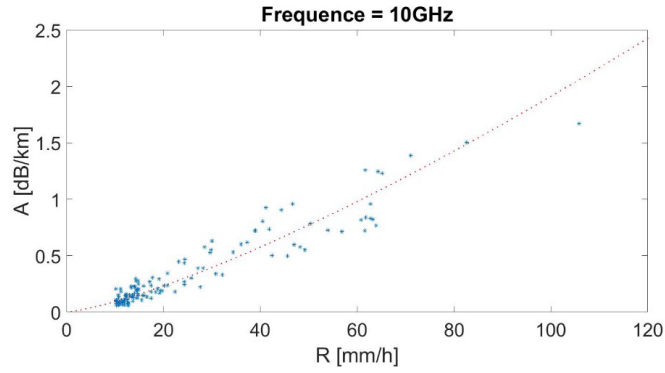


Figure 5. The same as Fig. 4. For the red power law regression curve the rain rate values above 10 mm/h were used.

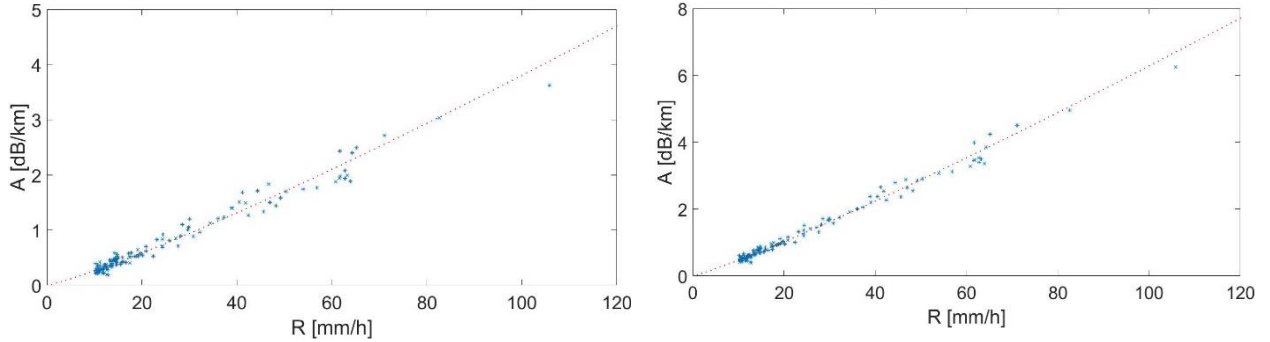


Figure 6.  $A$ - $R$  scatterplot for 15 GHz (left) and 20 GHz (right). Red dotted curve is the power-law approximation.

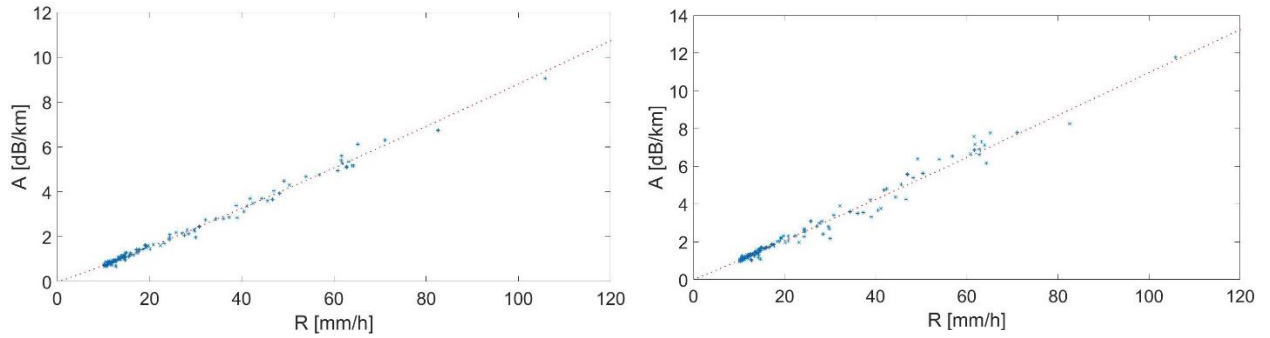


Figure 7.  $A$ - $R$  scatterplot for 25 GHz (left) and 30 GHz (right). Red dotted curve is the power-law approximation.

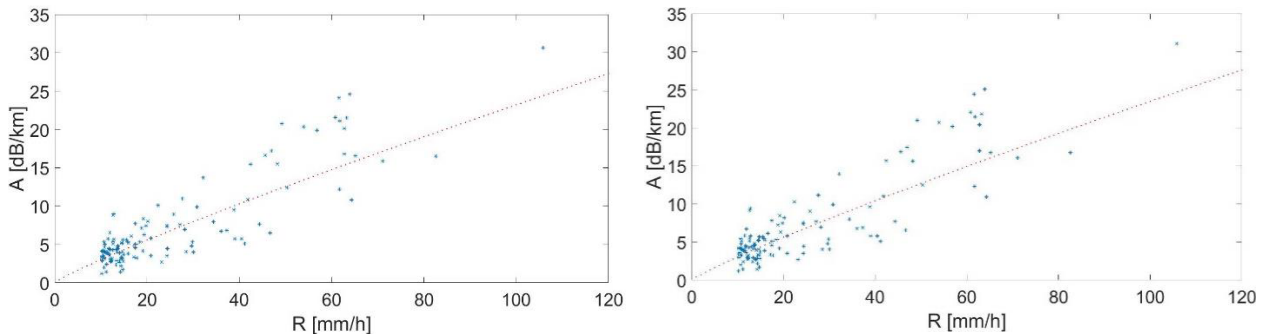


Figure 8.  $A$ - $R$  scatterplot for 95 GHz (left) and 100 GHz (right). Red dotted curve is the power-law approximation.

standard deviation of the specific rain attenuation values in 5 mm/h rain rate intervals (for instance, standard deviation values in intervals 10–15 mm/h ( $\sigma_{10,15}$ ), 15–20 mm/h ( $\sigma_{15,20}$ ) etc.. All ‘local’

standard deviations are averaged ( $\sigma_{MEAN}$ ) for pertinent frequency (Table 1). See Fig. 9

We used also the power-law approximation:

$$A \approx a \cdot R^b, \quad (6)$$

where  $A$  is attenuation,  $R$  is the rain rate.

Through the regression we found the  $a$ ,  $b$  values (Table 1) where the coefficient of determination is also shown. This coefficient is also a measure of the unambiguity of the experimental  $A$ - $R$  relationship.

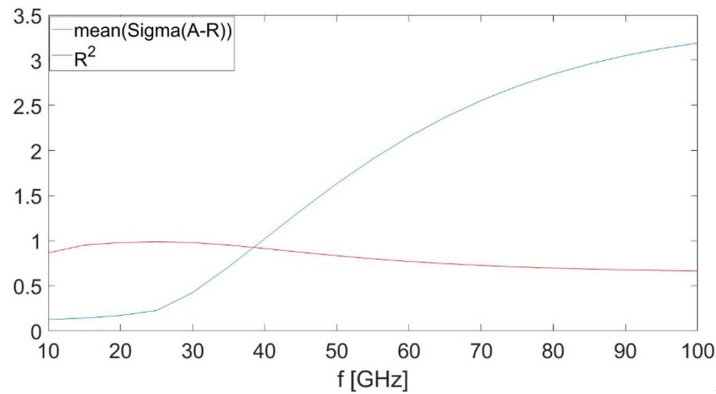


Figure 9. Standard deviation and coefficient of determination as a function of frequency. These parameters are showing unambiguity of  $A$ - $R$  relation.

Table 1. Standard deviation of unambiguity for 1) all  $R$  values 2)  $R$  values above 10 mm/h 3) coeff of determination for all  $R$  values and for  $R$  values above 10 mm/h 4)  $a$  and  $b$  values for the AR power law approximation.

f [GHz]	$\sigma_{MEAN}$ [dB/km] All R values	$\sigma_{MEAN}$ [dB/km] Reduced R values	$R^2$ All R values	$R^2$ Reduced R values	a Reduced R values	b Reduced R values
10	0.1064	0.1258	0.9465	0.8647	0.0046	1.3093
15	0.1229	0.1419	0.9556	0.9515	0.0181	1.1613
20	0.1503	0.1706	0.9676	0.9788	0.0353	1.1252
25	0.2001	0.2251	0.9766	0.9882	0.0595	1.0852
30	0.3693	0.4243	0.9820	0.979	0.0930	1.0358
35	0.6101	0.7093	0.9852	0.9515	0.1327	0.9922
40	0.8739	1.0208	0.9866	0.913	0.1736	0.9602
45	1.1401	1.3337	0.9860	0.8718	0.2116	0.9384
50	1.3952	1.6323	0.9831	0.8331	0.2455	0.9234
55	1.6310	1.9068	0.9772	0.7989	0.2749	0.9127
60	1.8432	2.1524	0.9683	0.7699	0.3002	0.9049
65	2.0304	2.3678	0.9565	0.7456	0.3217	0.8991
70	2.1930	2.5537	0.9423	0.7257	0.3398	0.8948
75	2.3325	2.7123	0.9266	0.7093	0.3548	0.8917
80	2.4510	2.8462	0.9101	0.6959	0.3670	0.8895
85	2.5509	2.9584	0.8936	0.6852	0.3768	0.8881
90	2.6344	3.0519	0.8777	0.6768	0.3844	0.8874
95	2.7035	3.1290	0.8630	0.6703	0.3902	0.8872
100	2.7601	3.1918	0.8496	0.6654	0.3943	0.8875

### 3.2. Specific Rain Attenuation $A$ — Radar Reflectivity Factor $Z$ Dependence ( $A$ - $Z$ )

Using Equations (2), (3) and (4) we prepared scatterplots of the specific rain attenuation — radar reflectivity factor dependence ( $A$ - $Z$ ). We can summarize that the unambiguity behaves like in the specific attenuation case. The unambiguity is minimal also for the 25 GHz frequency. Selected results are in Figs. 10–11.

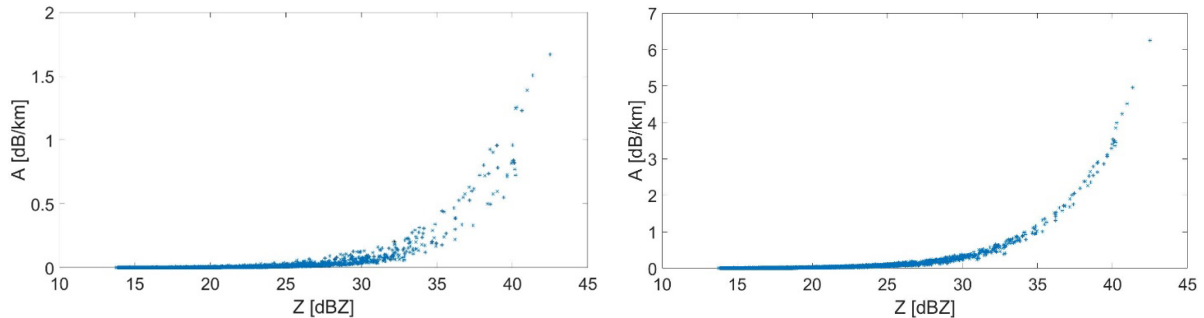


Figure 10. Attenuation-radar reflectivity factor scatterplot for 10 GHz (left) and 20 GHz (right).

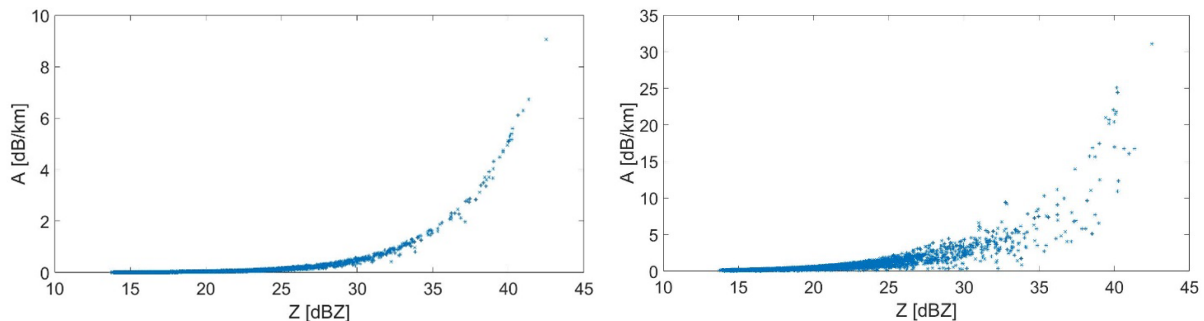


Figure 11. Attenuation-radar reflectivity factor scatterplot for 25 GHz (left) and 100 GHz (right).

In Table 2 we can see the dependence of standard deviation of interval values (10–15, 15–20 mm/h etc.) of specific rain attenuation on frequency.

Table 2. Standard deviation of 1 dB interval values (10–11 dBz 11–12 dBz etc.) of specific rain attenuation  $A$  for  $A$ - $Z$  relation.

$f$ [GHz]	10	15	20	25	30	35	40	45	50
$\sigma_{\text{MEAN}}$ [dB/km]	0.0442	0.0536	0.0692	0.0884	0.1445	0.2324	0.3347	0.4414	0.5476
	<b>55</b>	<b>60</b>	<b>65</b>	<b>70</b>	<b>75</b>	<b>80</b>	<b>85</b>	<b>90</b>	<b>95</b>
	0.6499	0.7463	0.8354	0.9163	0.9889	1.0533	1.1099	1.1590	1.09

## 4. CONCLUSION

We evaluated one minute DSDs from 2 year videodistrometer measurements in Prague. To each minute we assigned rain rate  $R$ , radar reflectivity  $Z$ , kinetic energy of rain  $E_k$  and specific rain attenuation  $A$  for frequencies 10–100 GHz. These quantities were computed from DSDs. From  $E_k$ - $R$  and  $E_k$ - $Z$  relations it is obvious that  $E_k$ - $R$  dependence is ambiguous while in the  $E_k$ - $Z$  dependence seems to be of power-law type. In the case of specific rain attenuation  $A$  we were curious of the unambiguity — frequency dependence. We discovered that in both cases of  $A$ - $R$  and  $A$ - $Z$  relationships the unambiguity strongly depends on the frequency of the microwave link. In the first case of the  $A$ - $R$  dependence the best unambiguity was found in the case of 25 GHz frequency. This unambiguity is decreasing toward either higher or lower frequencies. The same could be concluded for the  $A$ - $Z$  relation. As the measure of the unambiguity the standard deviation

of attenuation values was chosen. But the standard deviation is small in cases of small quantity value (specific rain attenuation). Therefore it must be replaced by relative standard deviation in the future. Therefore we approximated the  $A$ - $R$  relation by power law formula, the parameters  $a$  and  $b$  were found through the regression technique. And coefficient of determination serves as the measure of the unambiguity. This coefficient was maximal in the case of 25 GHz in both  $AR$  and  $A$ - $Z$  relationships

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