NEW FUZZY MULTIPLE CRITERIA EVALUATION METHOD AS A SUPPORT FOR INVESTMENT DECISION MAKING UNDER UNCERTAINTY

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Abstract: *This article is motivated by real investment decision making for which uncertainty is so typical. The uncertainty can be represented by an unstable development of some characteristics of the investment instruments, or vague investor's preferences. Moreover, the investment decision is usually made based on several criteria. To evaluate the investment instruments in a satisfactory manner, or namely to select suitable investment instrument(s), all mentioned aspects should be considered. For this purpose, a complex support tool is proposed in this paper. This fuzzy multiple criteria evaluation method uses triangular fuzzy numbers to express the element of uncertainty. It accepts a linguistically expressed importance of criteria which is very comfortable for a decision maker (investor). Linguistic terms are also transformed to the triangular fuzzy numbers based on the specified fuzzy scale. Using the concept of fuzzy sets (fuzzy numbers) provides a necessary quantification of the elements of uncertainty. The proposed method can select 'the best' alternative (i.e. investment instrument), to make a full ranking as well. All benefits of this fuzzy evaluation approach are illustratively demonstrated on a selected real investment decision making problem on the capital market with open unit trusts that are increasing in popularity within the Czech Republic.*

Keywords: *Investment Decision Making, Open Unit Trust, Triangular Fuzzy Number, Uncertainty, Vague Preferences.*

JEL Classification: *C44, G11.*

Introduction

Many people try to evaluate their free funds. One of the options is an investment. Selecting suitable investment instruments is not an easy task. It is usually influenced by several factors. Moreover, some factors can be in an uncertain form. Such a typical element of uncertainty on the capital market is the return of the investment instruments. It is often very unstable in time. Further, the investor's preferences can be also expressed inaccurately. For instance, the importance of the evaluative criteria can be expressed linguistically which is the user-friendly way for a (potential) investor.

The decision making theory provides many multi-criteria evaluation methods that are more or less suitable for a selecting, or evaluating investment instruments. Selecting a suitable method is namely determined by the character of a real decision making problem. This article deals with an increasingly popular investment in open unit trusts. Thus, the main aim is to solve the so-often real-life problem of a selection of suitable open unit trust(s) for an investment. It is demonstrated that principles and techniques of a decision making theory, or a fuzzy set theory, can be applicable for such a situation very well, although it is not used much in a real world. In practice, fundamental or technical analysis is sometimes applied (Murphy, 1999). The selection can be made via some quantitative or graphical indicator. The main disadvantage of these concepts is actually only one-criterion perspective (return, price, etc.). Another approach can be based on a human intuition which judges the investment instruments from one or also multiple perspectives. On the other side, the investment decision then would be fully not exact. The results of such methods are not complex and representative for so important (investment) decision.

Further, our investment situation is specific in the following matters. At first, the form of data is combined. It means that some data are crisp, while other data are vague. Another important fact is that a potential investor is able to express the criteria importance ‘only’ linguistically. If the investor is not so educated about this issue, then it is better that any additional information from him or her is not required. Further, ‘the best’ open unit trust is primarily selected. A full ranking of investment alternatives can be some interesting additional information for the investor. The investor is actually a layperson in this issue. However, the investor is glad when the procedure, based on which results he or she decides, at least basically understands. Therefore, having a method that is applicable and user-friendly is also important. This is a typical investment situation (not only) with open unit trusts. Many (multiple criteria) methods are not able to accept a combination of crisp and vague data. Other group of methods require some additional information (e.g. thresholds, α-level, etc.) which can be problematic for a decision maker. Some methods, however, are so complicated and hardly applicable. If no existing method does not enable to simultaneously fulfil all mentioned requirements, then a new one (or modification of the current concepts) must be proposed to solve the problem satisfactorily. This is a significant motivation for this article.

As we can see above, the quantitative methods can be more or less unsatisfactory for a particular investment decision making situation. To fulfil all requirements and circumstances of the investment situation, a new supporting decision making method must be proposed. The developed fuzzy multicriteria evaluation method is based on fuzzy preference relations and minimising distance from the ideal alternative principles. The algorithm is partly inspired by ELECTRE methods. It also uses McCahone’s approach and Hamming distance which are properly improved for their more comfortable applicability. All vague elements are expressed as triangular fuzzy numbers. Such a method enables a selection of suitable open unit trust(s) under uncertainty for a typical client of the bank Česká spořitelna.

This paper is organised as follows. After the introduction, the motivation to a new method is introduced, in addition to a related review of the literature with current multi-criteria evaluation methods. The algorithm of the proposed method is described in detail. Then the proposed method is applied to a real investment decision making situation on the capital market with open unit trusts. Finally, the article is summarised and some ideas for future research are outlined.

1 Fuzzy multiple criteria evaluation method

To comprehend all aspects of the proposed fuzzy method, the basics of fuzzy set theory must be presented. Then the algorithm of the fuzzy multiple criteria evaluation method can be described in detail.

1.1 Basics of fuzzy set theory

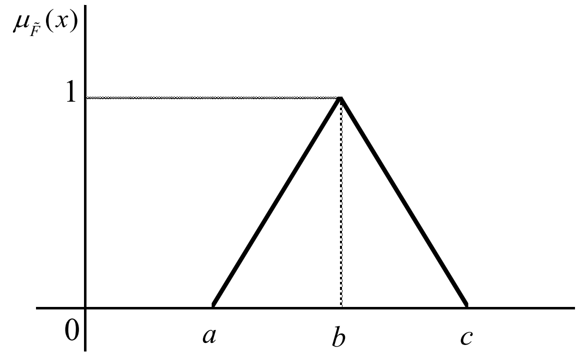
The following definitions can be seen in (Gupta and Bhattacharjee, 2010; Novák, 2000; Zadeh, 1965).

Definition 1 Fuzzy set is such a set whose elements are included with a certain grade of membership.

Definition 2 Membership function measures the grade of a set membership. It takes the value from interval $\langle 0,1 \rangle$. The higher value denotes the higher degree of the set membership.

The triangular membership function of the fuzzy number $\tilde{F}$ can be illustrated (on the left side) and mathematically formalized (on the right side) as follows

Fig. 1: Membership function of the triangular fuzzy number



$$\mu_{\tilde{F}}(x) = \begin{cases} 0 & x < a \wedge x > c \\ \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 1 & x = b \end{cases} \quad (1)$$

Source: Novák (2000)

Parameter *a* can be interpreted as the lower bound, *b* as the peak point and *c* as the upper bound of the fuzzy number. Mostly the position of parameters *a* and *c* is symmetric around the value of *b*. In other words, the membership function usually creates an isosceles triangle. Of course, different cases of the dissimilar sides of the triangle can be defined. The triangular fuzzy number can be formally written as follows $\tilde{F} = (a,b,c)$. The membership function is piecewise linear, so the computational operations with this type of fuzzy numbers are known and quite simple. They are able to express or approximate the vague decision maker's preferences or unstable input data very well.

Definition 3 Triangular fuzzy number is a convex fuzzy set with a triangular shape of the membership function.

1.2 Overview of the current fuzzy multiple criteria evaluation methods

We have many fuzzy multiple criteria evaluation methods potentially usable for a selection of the open unit trusts. Firstly, many methods are fuzzified in order to solve the multicriteria decision making problem under uncertainty. For instance, the Weighted Sum Approach (WSA) was modified to the form with criteria values and weights as fuzzy numbers (Baas and Kwakernaak, 1977; Bector et al., 2002). Similarly, the Analytic Hierarchy Process (AHP), proposed by Saaty (1977; 1978), is also fuzzified where decision making preferences about the relations between alternatives by the particular criterion are expressed as the fuzzy numbers, as well as the criteria weights (Buckley 1985; Laarhoven and Pedrycz, 1983). We know many other modifications of the AHP method, for example (Chen, 1997; Yu, 2002).

The conjunctive and disjunctive method was also fuzzified (Hwang and Yoon, 1981). These approaches were extended by vague, fuzzy criteria values and fuzzy threshold values. For this purpose, the trapezoidal fuzzy numbers are applied. The comparison of fuzzy evaluations is made via the concept of possibility and necessity (Dubois et al., 1988).

Another well-known multicriteria evaluation method is the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), whose computational principle is based on the minimisation of a distance from the ideal alternative (Hwang and Yoon, 1981). We have many fuzzy modifications of this method in order to solve more real decision-making situations. These versions differ in the type of the used fuzzy number, the crisp or fuzzy weights or the method for fuzzy number comparison. More information about particular modifications can be found in various studies (e.g. Chen and Hwang, 1992; Chu, 2002; Wang and Elhag, 2006). The summary of the method modifications is in the publication of Kahraman (2008).

The large group of multiple criteria evaluation methods comprise approaches based on the preference relation principle (i.e. outranking methods). The pioneer is the ELimination Et Choix Traduisant la REalité (ELECTRE) I developed by (Roy, 1968). During the 1070s, this group of methods expanded, for instance, with either ELECTRE II (Roy and Bertier, 1973) or ELECTRE III (Roy, 1978). Further, the method Multicriterion Analysis of Preferences by means of Pairwise Alternatives and Criterion comparisons (MAPPACC) was also proposed (Matarazzo, 1986). However, step by step, the concept of fuzzy preference relations was discussed. The first mentions are in Roy (1977) and Roy (1980). Other concepts are, for instance, mentioned in (Martel and D'Avignon, 1986; Siskos et al., 1984; Takeda 1982). These concepts consider a different form for a specification of fuzzy relations. The other approaches include vague input data expressed by fuzzy numbers (Gheorghe, 2005; Wang, 2001).

1.3 Incentives for designing own evaluation method

Because we solve a particular real decision making situation, it is sometimes not possible to apply some of the existing methods because it is not able to take into account all typical circumstances for a solved problem. Then, we propose an evaluation algorithm 'bespoke', which fulfils all our conditions and premises. This is absolutely standard (inevitable) procedure, as we can see (e.g.) by Hua et al. (2005) in green manufacturing or Omero et al. (2005) in the field of performance assessment.

Now let us introduce the reasons and ideas for a proposal of a new fuzzy multicriteria evaluation method in greater detail. At first, the capital market is an unstable environment. Then some elements of uncertainty (or proximity) should be included. Concretely, the input data can be vague, as well as the decision maker's preferences. They can be expressed in a linguistic form. To quantify these elements, a concept of fuzzy sets (i.e. fuzzy numbers) can be very well used. Then an interest in the group of fuzzy multicriteria evaluation methods is natural.

Secondly, many fuzzy multiple criteria evaluation methods require all input data in vague (fuzzy) form as Baas and Kwakernaak (1977), Dubois and Prade (1982), Laarhoven and Pedrycz (1983), Ling (2006) and the others to the present. However, some data can be crisp (in the strict, purely quantitative form). In this case, there is no room for any vagueness or a measure of subjectivity. Then it is irrational to transform a strict element into the fuzzy form.

Further, some methods (namely outranking methods) require some additional information from a decision maker. It is usually a value of threshold(s) or α-level. A determination of these values can be very difficult for a decision maker. In addition, many methods (WSA, TOPSIS, etc.) require a normalisation of the original input data. This process can negatively influence the results because the original values may be distorted. Further, an evaluation principle based on the utility function can be problematic, thanks to a construction of the membership function of the aggregate utility of particular alternatives. This problem namely occurs in the situation when the membership functions of a partial utility of a concrete alternative are not piecewise continuous, differentiable functions. Then it is better to apply another principle.

However, it is necessary to focus on the form of the results. Methods using a minimization distance from an ideal alternative usually provide a full ranking of alternatives. On the other side, the outranking methods can divide the alternative into 'good' and 'bad' ones, or according to a similar key. However, in our investment case, a selection of the compromise ('the best') alternative is mainly needed. The quotation marks are used because it is deceptive to talk about the best alternative in the multiple criteria decision making problem. We usually do not have the alternative with the best value of all evaluative criteria. The full ranking of alternatives is not necessary. Of course, a ranking can be required in special cases. Then a simple additional procedure can be integrated.

Yet, many methods are computationally difficult, see Baas and Kwakernaak (1977), Buckley (1984), or Buckley (1985). We tried to propose a method whose algorithm is computationally quite simple and is at least basically comprehensible for decision makers (i.e. investors). It is not possible to suppose that the algorithm is based on some trivial calculation. Moreover, today's computer technology can help very much to manage difficulties in a calculation process. For the practical usage of the method, the comprehensibility of the principles of a decision making process is very essential. Then the results are more plausible for a decision maker.

Let us summarise the main aspects that lead to a new fuzzy multiple criteria evaluation method. The method is able to select only one, i.e. 'the best' alternative. Some criteria values should be strict, while other criteria values should be vague. The importance of the criteria is expressed linguistically by a decision maker. The algorithm transforms these expressions into a strict quantitative form. Normalisation of the criteria values and a transformation of minimising criteria to a maximising form are not required. The algorithm should not be computationally difficult, and it is user-friendly. This combination of the conditions and requirements is unique (compared to the facts mentioned above), a new method must be proposed. This method is inspired by some principles of the ELECTRE methods. For evaluation, the combination of (fuzzy) preference relation and minimising distance from the ideal alternative principles are integrated. For comparison of the fuzzy numbers, the modified McCahone's approach is applied. The distance between fuzzy numbers is measured by the Hamming distance (Hamming, 1950), which is modified to eliminate its weakness.

1.4 Algorithm of the proposed method

A new fuzzy multiple criteria evaluation method will be described in terms of the several following steps.

Step 1: Given the matrix $\mathbf{Y} = (y_{ij})$, where y_{ij} $(i = 1,2,...,p;\ j = 1,2,...,k)$ represents (crisp, vague) valuation of the i-th alternative by the j-th criterion (criteria values). The vague evaluation is expressed as a triangular fuzzy number, or a fuzzy number with a triangular membership function as follows

$$\tilde{F}_X = (x_{\min}, \overline{x}, x_{\max}), \tag{2}$$

where $\overline{x}$ is a mean and $x_{\min}$, or $x_{\max}$, denotes a minimum, or maximum, of all values of a vague element (criterion) X.

An importance of the particular criteria is stated linguistically. These linguistic expressions are transformed into the triangular fuzzy numbers in the interval $\langle 0,1 \rangle$. The strict weights are formulated via the optimisation model based on the maximin principle (Borovička, 2014a). Then the vector of crisp weights is marked as $\mathbf{v} = (v_1, v_2, ..., v_k)^T$, where v_j $(j = 1,2,...,k)$ is the weight of the j-th criterion.

Step 2: In the second step, we must find out the definite ranking for all alternatives according to each criterion. For the strict criteria values, it is trivial. It is more difficult in the case of fuzzy numbers. For this procedure, McCahone's approach (McCahone, 1987) is applied with some smaller necessary modifications and improvements, as proposed in (Borovička, 2014b).

Step 3: We specify the set $I_{iRj} = \left\{ r \mid y_{ir} > y_{jr};\ i, j = 1, 2, ..., p; i \neq j \right\}$ containing the indices of criteria r according to which the alternative i is evaluated better than the alternative j. Then we can formulate (as in ELECTRE III) the matrix $\mathbf{S} = (s_{ij})$ as follows

$$\begin{aligned} s_{ij} &= \sum_{q \in I_{iRj}} v_q \qquad i, j = 1,2,...,p,\ i \neq j, \\ s_{ij} &= \times \qquad i, j = 1,2,...,p,\ i = j. \end{aligned} \tag{3}$$

The element of the matrix s_{ij} may be interpreted as the volume of the preference of the i-th alternative over the j-th alternative. For each i-th alternative, the average preference relation is calculated as follows

$$s_i^{average} = \frac{\sum\limits_{\substack{j=1 \\ i \neq j}}^{p} s_{ij}}{p-1} \qquad i = 1,2,...,p. \tag{4}$$

Finally, the threshold value is computed by the following formula

$$s = \frac{\sum\limits_{\substack{j=1 \\ i \neq j}}^{p} s_{ij}}{p(p-1)} = \frac{\sum\limits_{i=1}^{p} s_i^{average}}{p}. \tag{5}$$

Step 4: First, we choose the alternatives that satisfy the following formula

$$s_i^{average} \geq s \,. \tag{6}$$

Secondly, the distance from the ideal alternative is computed. In the case of crisp valuation y_{ij} $(i = 1, 2, ..., p; j = 1, 2, ..., k)$, the distance from the best value by the j-th maximizing criterion is as follows

$$d_{ij\_\max}^{crisp} = H_{j\_\max}^{crisp} - y_{ij\_\max}^{crisp}, \tag{7}$$

where $H_{j\_\max}^{crisp} = \max_i (y_{ij\_\max}^{crisp})$ is the best valuation according to the j-th criterion.

The distance from the best value by the j-th minimising criterion is calculated as

$$d_{ij\_\min}^{crisp} = y_{ij\_\min}^{crisp} - H_{j\_\min}^{crisp}, \tag{8}$$

where $H_{j\_\min}^{crisp} = \min_i (y_{ij\_\min}^{crisp})$ is the best valuation according to j-th criterion.

If the criteria values are the fuzzy numbers, then we use the modified concept of the Hamming distance to find out the distance from the best valuation. The Hamming distance between two fuzzy numbers F_i and F_j is defined as (Lai and Hwang, 1994)

$$d(F_i, F_j) = \int_{-\infty}^{+\infty} |\, \mu_{F_i}(x) - \mu_{F_j}(x) \,|\, dx \;. \tag{9}$$

This concept sometimes ignores a location of the fuzzy numbers on the horizontal axis. To calculate a more representative distance[1] from the best value, we propose to distinguish two parts: $\mu_{F_i}(x) \geq \mu_{F_j}(x)$ and $\mu_{F_i}(x) < \mu_{F_j}(x)$. Let us denote fuzzy number F_j as the best valuation and F_i as a particular valuation whose distance from the best one is calculated. Specify

$$X^1 = \left\{x; \mu_{F_i}(x) \geq \mu_{F_j}(x)\right\}, \quad X^2 = \left\{x; \mu_{F_i}(x) < \mu_{F_j}(x)\right\}. \tag{10}$$

Then, the distance between F_i and the best valuation F_j is formulated for the maximising criterion as follows

$$d(F_i, F_j) = \int_{X^2} |\, \mu_{F_i}(x) - \mu_{F_j}(x) \,|\, dx - \int_{X^1} |\, \mu_{F_i}(x) - \mu_{F_j}(x) \,|\, dx \,. \tag{11}$$

The distance between F_i and the best valuation F_j is formulated for the minimizing criterion as follows

$$d(F_i, F_j) = \int_{X^1} |\, \mu_{F_i}(x) - \mu_{F_j}(x) \,|\, dx - \int_{X^2} |\, \mu_{F_i}(x) - \mu_{F_j}(x) \,|\, dx \,. \tag{12}$$

In the case of the fuzzy valuation, we should mark d_{ij}^{fuzzy} as the distance of the i-th alternative from the best valuation by the j-th criterion that is computed via the previous formula. Finally, the general distance from the ideal alternative for the i-th chosen alternative is computed as follows

[1] It could be called as 'pseudodistance' because this rate is no distance in the true sense. However, it is a better rate for a quantitative comparison of fuzzy numbers.

$$d_i = \sum_{j=1}^{k} \left[\frac{d_{ij}^{nonfuzzy}}{\max_i (d_{ij}^{nonfuzzy})} + \frac{d_{ij}^{fuzzy}}{\max_i (d_{ij}^{fuzzy})} \right]. \quad (13)$$

The partial distance by each criterion j is standardised with the view of their comparability. The alternative with the smallest distance from the ideal alternative is chosen as 'the best' (i.e. the compromise alternative). Of course, alternatives can be ranked according to increasing distance from the ideal alternative.

The compromise alternative is non-dominated. It is no doubt that it is not possible to choose the dominated alternative without the non-dominated alternative which dominates it by the preference relations procedure, thus, before the evaluation by distance from the ideal alternative. If some chosen alternative is dominated, it has no chance to be selected in the second phase of a measurement of distance from the ideal alternative computation because the alternative that dominates it must have a shorter distance from the ideal alternative.

2 Selection of suitable open unit trust(s) for investment

A potential investor decided to invest his or her free financial resources in an open unit trust offered by Česká spořitelna. Recently, investing in open unit trusts has become more and more popular in the Czech Republic. This bank was selected because of its strong position on the capital market with open unit trusts. The other motivation behind the selection was my long personal experience with its products. The open unit trusts can be divided into three basic groups – mixed, bond and stock. For a rather illustrative instance, a group of bond funds was analysed in this article. Thus, we have five bond open unit trusts – Sporoinvest, Sporobond, Trendbond, Korporátní dluhopisový and High Yield dluhopisový.

There is no doubt that the investment decision is influenced by many factors. The investor evaluates the investment alternative according to three main criteria – return, risk and costs. The return of open unit trust is created by earnings from interest-bearing securities and capital profit (loss). Both items are capitalised in a fund price. Then, an open unit trust return is comprehended as a difference in the purchase and the particular selling price (Steigauf, 2003). The return of the shares fund is determinate as a monthly of the time period from 2009 to 2017. Even less liquidity of the open unit trusts, the investment is comprehended as rather longer-time. The historical period is selected to best reflect a longer development of the funds' prices and returns, respectively. The next evaluative criterion is risk. It can be comprehended as the investor's apprehension about investment loss (Boroson, 1997). In other words, a potential investor is afraid that his or her investment aims will not be fulfilled, and he or she will not achieve expected returns. The risk of open unit trust is standardly stated as a variance, or a standard deviation (Markowitz, 1952) of a fund return rate of monthly returns in the period mentioned above. The cost connected with a shares fund is represented by duties, namely the initial charge and management fee. The initial charge is subtracted from the stake, and the management fee is subtracted from the fund property (Ježek, 2002). As mentioned above, the cost can be divided into two main charges. However, there are a few other fees that are usually unknown to an investor, such as an investment advisor charge, spending on an audit and auditing company, factorial or license costs. All expenses (without an initial charge) connected with the investment in the shares fund are implicated in a Total Expense Ratio (TER) (Středoevropské centrum pro finance a management, 2018). This indicator shows the volume of all expenses to the total share fund property (Ježek, 2002). For a practical application, TER values (as well as the initial charges) are available for the year 2017. Of course, other characteristics can be included in the analysis, for example, currency, locality, fund credibility, fund management and mood on the capital market. However, these criteria rather determine an initial selection of open unit trusts. Then the multicriteria evaluation is explicitly not influenced by these characteristics. Moreover, it is obvious (also according to my long-term investment experiences) that the three mentioned characteristics are the most essential for a potential investor.

A multi-criteria analysis is supported by the following software. Matlab was used for integral calculations. Other supporting computation operations were made in MS Excel. The optimisation model was solved in the Lingo optimisation software.

The return, as an unstable element, is expressed in the fuzzy form according to (2). Risk and cost are presented by the crisp value. The data about the return, risk and cost are computed in the following table (Tab. 1).

Tab. 1: Data of bond open unit trusts

Open unit trust	**Return [%]**	**Risk [%]**	**Cost [%]**
Sporoinvest	(-1.07, 0.11, 1.33)	0.38	1.05
Sporobond	(-3.51, 0.35, 4.61)	1.29	2.05
Trendbond	(-5.92, 0.22, 6.48)	2.02	2.69
Korporátní dluhopisový	(-9.45, 0.43, 10.48)	3.31	2.69
High Yield dluhopisový	(-12.95, 0.38, 13.99)	4.43	2.51

Source: Investiční centrum České spořitelny (2017) and own calculation

The preferences about criteria importance are described linguistically by the investor: risk – *very important*, return – *important*, cost – *little important*. According to my investment experiences, this is a typical investment strategy. Return is understandably important. However, most investors in open unit trusts prefer a more stable investment, with a lower possibility of loss. Thus, the risk is slightly more important than the return. This fact also determines a selection just from the group of bond open unit trusts. A lower importance of cost is also caused by insufficient knowledge. Investment consultants do not talk much about the cost aspect of the investment.

According to the transformation procedure proposed by Borovička (2014a), the aforementioned linguistic terms about importance of the evaluating criteria are expressed as triangular fuzzy numbers from a pre-specified 5-stages fuzzy scale. Then their strict form is computed via a model of linear mathematical programming. All necessary data and results are shown in Tab. 2.

Tab. 2: Criteria importance in a cardinal form - weights

Criterion	Fuzzy number	Weight
Risk	(0.28, 0.33, 0.38)	0.55
Return	(0.45, 0.55, 0.65)	0.3
Cost	(0.13, 0.16, 0.19)	0.15

Source: own calculation

Sporoinvest was the best in risk and cost criteria. According to the modified McCahone's approach, Sporobond is the best in return. To choose 'the best' alternative, the proposed fuzzy method is employed. The grade (volume) of preference (3) is calculated for each couple of alternatives. Before measuring distance from the ideal alternative, two funds were chosen according to the rule (6) based on (4) and (5) – Sporoinvest and Sporobond. Sporoinvest had the shortest distance (13) from the ideal alternative (equal to 1 vs. 2 for Sporobond), so it is the compromise alternative. Anyway, it was quite expectable, because it dominates in the most (and least) important criterion. If such weight was on the return, then the 'winner' would be Sporobond.

2.1 Discussion of the results and the application benefits

This methodical approach can be applied in each group of open unit trusts and then the final investment portfolio can be created by appropriate methods that are based on the mathematical programming (Borovička, 2012). In this way, the investor can focus on a larger number of funds or divide the funds between 'bad' and 'good'. Potential investors can be well versed in the large group(s) of open unit trusts. This is supporting information for the following process of a portfolio making which potentially becomes easier.

This real decision making situation illustratively shows several application benefits of the proposed fuzzy approach. The first benefit is a possibility of quantitative expression of the vague element (as fuzzy number (2)) or preferences that are typical for a decision making on the capital market. It turns out that a possibility of a unique combination of crisp and fuzzy data is crucial for an investment decision making. The second benefit is a user-friendliness of the proposed procedure, which does not require any additional information from a decision maker (e.g. the threshold value is 'self-calculated' by (5)). The preferences about the criteria importance can be expressed 'only' linguistically that is very comfortable for a decision maker. The proposed method then enables a various investment strategy which makes it broader and more complex usable. As can be seen, the method is able to consider various qualitative and quantitative criteria that can influence the investment decision. This combination is unique compared to the current approach. Moreover, such a complex support tool can be applied to other decision making situations in the environment of uncertainty (e.g. selecting a suitable project, bank product or financial asset).

Conclusion

The main contribution of this paper is a proposal of a fuzzy multiple criteria evaluation method that is primarily adapted to an investment decision making problem on the capital market (with open unit trusts) under uncertainty. The vague elements (i.e. criteria values or the decision maker's preferences) are represented by triangular fuzzy numbers. This approach is a complex support tool for evaluating investment instruments, or selecting suitable ones. The method accepts the multi-criteria analysis, the uncertain input data or the linguistically expressed importance of evaluative criteria. These benefits make the approach user-friendly compared to other current methods. These benefits, and the power of such a method, are demonstrated on today's very common investment situation describing a selection of the suitable open unit trusts (offered by Česká spořitelna).

Future research could be devoted to possible modifications of the proposed method. One of them is an application of a different type of fuzzy number (e.g. trapezoidal), if a real decision making problem would require. This is also related to the technique of comparing fuzzy numbers that could be slightly modified, or whether other approaches could be applied.

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