Determination of the geographical location of a central facility

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Abstract

Central facility location models are mostly used in logistics and transport networks to find the optimal position of a central warehouse, distribution centre or terminal. These models usually minimize the total transportation or distribution costs, which are based on the costs per unit, transport volumes and distances between existing customers and future center.
This paper analyses the different possibilities for determining the distances between customers and the central facility. The coordinates of existing facilities (customers) and a suitable coordinate system are the default inputs for solving. The distance can be expressed by different ways e.g. direct, corrected, rectangular or quadratic. This expression is usually sufficient for relative short distances (tens to hundreds km) in planar space. In the case of longer distances, the curvature of the earth´s surface should be considered. The paper deals with the possibility of using the geographical coordinates and orthodromic distance in location models. Authors focus on simple and practical solving with the use of commonly available software applications. The accuracy of the different distance calculations will be compared.
KEY WORDS: *facility location, geographical coordinates,*

1. Introduction

Location models are used for location of various objects considering the interaction with other objects and/or service of defined area.

Covering problems lie in the location of service centers for a given set of objects with minimal costs, i.e. the number of service centers should be minimized. The maximum distance to service center is given for every served object. This problem can be applied to optimal location of central rescue, fire or police stations. The given maximal driving time to all served objects shall be respected [4,5].

Center location problems lie in such service center location which minimizes the maximal (weighted) distance between every served object and the nearest service center. This problem can be applied to optimal location of central rescue, fire or police stations. The maximal driving time is not given but should be kept to a minimum.

Median location problems lie in such service center location which minimizes the sum of weighted distances between all served objects and the nearest service center. Served objects are rated at weights. Weight can represent the importance of the object or service requirements (e.g. required transport volumes). This problem can be applied to location of central warehouse, depot or logistic center based on minimization of total transportation costs. The problem is expressed by this function:

$$\min f(m) = \sum_{v} w(v) \cdot d(m,v) \tag{1}$$

where m - searched node in the graph $G=(V,H,c,w)$; $w(v)$ – weight of the node v, $d(m,v)$ - distance between m and v.

2. Definition of location area

The characteristics of location area are very important criteria for the definition of location problem. Problems can be divided into planar location problems, discrete location problems and network location problems.

Planar location is based on continuous location area. Service centers can be located anywhere in the defined geometric area. Fermat-Weber location problem which focuses on median location in Euclidean space is typical example of planar location. This problem is usually solved by Weiszfeld method. [2,4].

Each model of solving differs according to used metric. Taxicab geometry (manhattan distance, rectilinear distance) can be used to solve location problems in the city areas with rectangular street network or in industrial applications, e.g. for location of a machine in production hall. The distance d is:

$$d((x_1, y_1),(x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2| \tag{2}$$

where x_i,y_i – coordinates of the object i.

The direct distance in Euclidean metrics is:

$$d((x_1, y_1),(x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{3}$$

where x_i, y_i – coordinates of the object i

In this case, the distance is displayed as a straight line. It can be used e.g. for location of a distribution center, logistic center or depot. In the case of longer distances (thousands of kilometers), the curvature of the earth´s surface should be considered. Distance can be then expressed by spherical trigonometry. [3]. The Earth´s surface is displayed as a reference sphere with radius *R=6378,1 km* and the shortest distance *d* is displayed as an arc (orthodrome):

$$d((x_1, y_1),(x_2, y_2) = \arccos\left(\sin x_1 \sin x_2 + \cos x_1 \cos x_2 \cos\left(y_1 - y_2\right)\right) \cdot R \cdot \pi/180 \tag{4}$$

where x_1 , x_2 – object latitude [°], y_1 , y_2– object longitude [°], R – radius of the Earth [km]

If we search for service centers location for customers in the whole world, the distance on the ellipsoid could be more accurate. [1].

The distance on sphere or ellipsoid can be used directly for air transport or partly for maritime transport. Road, rail and inland waterway transportation are actually realized in the transport network. The geometrical distances of each object in the space are only approximations of real distances in the transport network. The question is when could be used the transport network model (graph) and when would be the approximation more efficient. Many practical tasks really should be solved by the transport network model. But the construction of such model can be very difficult and time-consuming. The difficulty increases with the number of differently located customers. The customers in practice often demands the fast and simple way of solving.

3. Formulation of the problem

Our objective is to find out how the different distance functions influence the location of central facility and how big is the difference between calculated distance and the driving distance in the real road network. We made the calculation for a set of fictive potential customers.

We solved the model example of one central facility location for the sample set of 39 randomly selected customers (cities) in Europe. We assumed their service by road transport. The distances between the customers varies from hundreds to thousands km. This model case is applicable to location of a logistic center, distribution center, terminal or one central warehouse for the customers with different transport volume requirements.

The main goal is to find the location with minimal total transportation costs. If we assume that the center facility can be located anywhere in the defined area, we can formulate the problem as median location in continuous space.

These inputs are required:

- location of each customer (geographical coordinates)
- requirements of each customer (e.g. transport volumes)
- transportation costs per unit and km.

4. Solution

We used Solver application in MS Excel. The constant weight of each customer (city) was determined according to the population. Starting center point was calculated using the Center of Gravity method.

The minimizing function in Euclidean metric (3) is:

$$\min f(x, y) = \sum_{i=1}^{m} w_i \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{5}$$

where x,y – coordinates of central facility, w_i – weight of customer i, x_i, y_i – coordinates of the customer i, m – the number of customers

Using the geographical coordinates in (5) will result the geographical coordinates of the central facility with minimal total transportation costs. The disadvantage lies in the impossibility of expressing distances between customers and central facility directly in km.

The minimizing function using spherical trigonometry (4) is:

$$\min f(x,y) = \sum_{i=1}^{m} w_i \cdot \arccos\left(\sin x \sin x_i + \cos x \cos x_i \cos\left(y - y_i\right)\right) \cdot R \cdot \pi/180 \tag{6}$$

where x – latitude of central facility [°], y – longitude of central facility [°], w_i – weight of customer i, x_i,y_i – coordinates of the customer I [°], m – the number of customers, R – radius of the Earth [km].

We also have determined the real driving distances between customers and central facility (using map server mapy.cz) to compare the solution with.

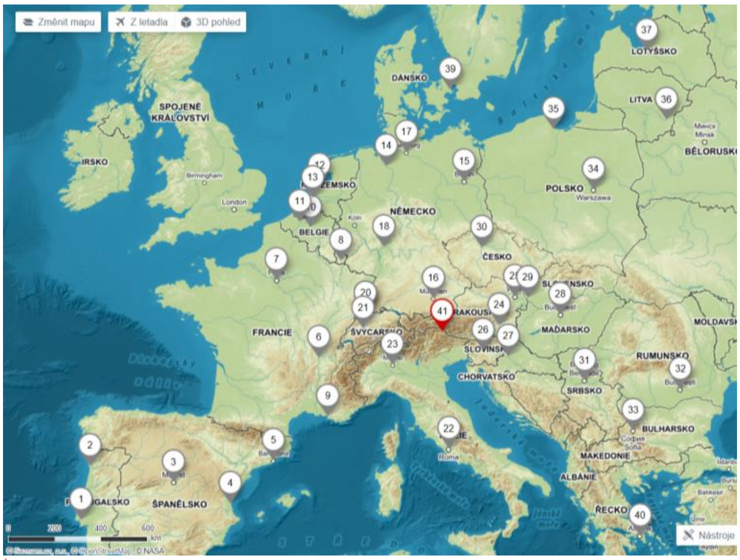
The sample of the solving is shown in the Figure 1.

1	City	Longitude	Latitude	w_i	d_2	d_{21}	d_{22}	N2	N3	d_{22}/d_2	d_{21}/d_2
2	Paris	2,35267550	48,8546710	21,38	733	890	822	15680,90	17574,36	1,1207	1,2135
3	Basel	7,60002047	47,5513984	1,68	352	464	402	591,04	675,36	1,1427	1,3189
4	Kopenhagen	12,58122057	55,6850819	5,03	844	1032	968	4245,13	4869,04	1,1470	1,2228
5	Lisboa	-9,14506674	38,7115362	5,17	2010	2501	2308	10394,23	11932,36	1,1480	1,2440
6	Bratislava	17,12956041	48,1529702	4,31	364	452	418	1567,25	1801,58	1,1495	1,2430
7	Wien	16,37150377	48,2079119	16,56	307	361	354	5089,51	5862,24	1,1518	1,1746
34	Praha	14,42692369	50,0868275	12,94	272	386	332	3522,50	4296,08	1,2196	1,4180
35	Ljubljana	14,50932115	46,0519651	2,78	286	360	355	796,16	986,90	1,2396	1,2570
36	Athens	23,74223170	37,9759042	30,58	1462	1988	1866	44720,27	57062,28	1,2760	1,3594
37	Roma	12,48783678	41,8999089	28,72	691	895	886	19851,40	25445,92	1,2818	1,2948
38	Zagreb	15,98698229	45,8074401	7,9	383	499	491	3025,86	3878,90	1,2819	1,3028
39	Marseilles	5,38407750	43,3084750	8,5	754	1035	968	6406,41	8228,00	1,2843	1,3732
40	Milano	9,18095201	45,4810126	13,24	374	539	482	4946,06	6381,68	1,2903	1,4428
41								**120836,7**	**148994,7**		
42	Center point	12,11758057	46,82026568								
43	Median (eucl.dist)	12,41676626	47,88663287								
44	Median (orthodrome)	12,23509829	48,10657963								
45	Median (cor. orthod.)	12,29208325	48,00172938								

Fig.1 Solution of the central facility location in MS Excel.

5. The results of the solution

The locations of the fictive customers are shown in the Fig. 2. The point 41 (Fig. 2) shows the starting center point calculated using the Center of Gravity method. The calculated location of the central facility using Euclidean metric and spherical trigonometry differs by about 30 km. The great circle distances (orthodrome) between central facility and customers vary from 49 to 2010 km (average 775 km). Only one customer (2,5%) is located within 100 km, 30% of customers is within 267-500 km, 38% of customers is within 500-1000 km and 23% is over 1000 km. From this point of view, the difference between the two used metrics is not very important.



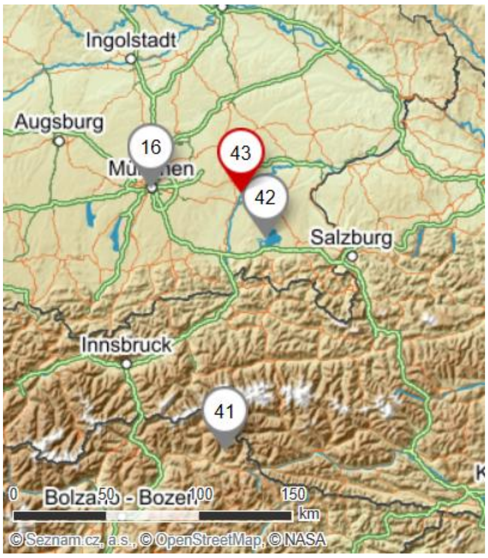


Fig.2 Set of customers with starting center

Fig. 3 Locations of central facility

The difference between calculated distances and the real driving distances on the road network will be more important for practice. We have determined the real distances for the shortest and the fastest route.

Differences between the shortest route and great circle distance vary between 12-28%. These differences point to significant importance of terrain on the route. All customers within interval of 20-28% are located beyond the Alps. The differences between the fastest route and great circle route are greater (17-44%). This is caused by choosing the routes outside the mountains and routes with higher driving speed. The real route chosen by carrier is very important. This decision-making process can be influenced by the established service model (time and distance criteria), character of the route (e.g. terrain) or by the costs (e.g. different fees, duty, taxes).

To refine the solution, we could use the corrected distance which considers the differences between theoretical solution and the usually used routes. We need to determine a suitable coefficient which can be based experimental on the real distance of the selected set of customers (columns d_{22}/d_2 and d_{21}/d_2 in the Fig.3). This coefficient can be used for customers in certain area with similar character.

6. Conclusions

The model example of central facility location in the area with diameter of about 2000 km shows that the differences between the solutions using different distance metrics are not very high. In our case, the corrected location of central facility is situated 18 km away from the calculated location (orthodrome). The difference is in the total transportation costs which can be the important economic indicator for the decision maker.

References

1. **Valencia, J.C.; Morales, J.O.C.; Flores, J.L.M.** 2016. Logistic model for the facility location problem on ellipsoids. International Journal of Engineering Business Management 8(49),1-9.
2. **Chandrasekaran, R;** Tamir. A. 1990. Algebraic Optimization: The Fermat-Weber Location Problem. Mathematical Programming 46, 219-224.
3. **Katz, I.N. and Cooper, L.** 1980. Optimal Location on a Sphere. Computers & Mathematics with Applications, 6, 175-196.
4. **Francis, R.; McGinnis, L.; White, J.** 1992. Facility Layout and Location: An Analytical Approach. New Jersey: Prentice-Hall, 444 p.
5. **Ledvinová, M.** 2013. Teorie dopravy. Pardubice: Univerzita Pardubice.