A NEW METHOD TO ESTIMATE LOAD CARRYING CAPACITY OF THIN-WALLED CONICAL SHELLS WITH CIRCUMFERENTIAL RING LOADED BY AXIAL FORCE

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Abstract

The conical shell with different shell thicknesses and base angles at the lower edge are investigated in the study. A new method is proposed to estimate load carrying capacity of the conical shell structures with a base angle less than 25°. The proposed method is also applicable to different radial stiffnesses of the structure. Empirical relationships are established based on the results of the numerical analysis.

Keywords

Conical shell, Circumferential ring, Load Carrying Capacity, Axial loading, FEM

1 INTRODUCTION

The load carrying capacity of a structure which is computed by merely linear buckling formulations is not a reliable way regarding safety aspects for nonstandard structures. These approaches may give higher loads than the real load carrying capacity. The additional bending effect occurs in nonstandard structural elements. For instance, a conical shell with base angle higher than 25° which is clamped on the radial direction (a standard structure) under uniform axial load has almost a clear membrane stress. However, a bending state occurs at the beginning of the loading from nature of the conical shell structure with the base angle less than 25° (nonstandard structure). For this reason, linear FEM analysis or theoretical calculations cannot be used to evaluate the load carrying capacity in nonstandard structural elements (conical shells with the base angle less than 25°, spherical cap, etc.). Thus, the loss of stability approach is a vital issue to simulate real system response under axial loading. Determining the load carrying capacity of the nonstandard structure might be infeasible by referring to the procedures within the context of the standards and recommendations because it is difficult to estimate the nonlinearity of the structure. Likewise, the recommendations and standard methods are based on the linear theory of the shells.

In present days, updated standards and recommendations provide useful approaches. They solve the stability of the conical shells with the base angle, $\propto_c$ (see Figure 1) which is higher than 25° and clamped lower end [1,2]. Nevertheless, the standard methods are not applicable to the shells which have the base angle less than 25°. Besides, the rules which are included in the

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recommendations can be applied only to conical shells which have clamped edges or edge with the very stiff ring. In other words, if a conical shell has either base angle less than 25° or free/flexible radial stiffness at the edges, these rules cannot be applied.

Stability of the thin-walled shell structures has been studied by many prominent authors. Results of their studies are embedded in standards, regulations, and recommendations. In one of the earliest research in the area, *Lackman and Penzien* experimentally investigated the buckling of conical shells under axial loading [3]. The conclusions were drawn on the suitability of the proposed equation, which includes a proper correction factor, to predict the critical buckling load of axially loaded conical shells. As one of the pioneers in the field, *Weingarten et al.* studied and discussed the stability of cylindrical and conical shells under axial loading. Both steel and Mylar polyester sheet materials were used for an extensive experimental programme [4]. *Singer* investigated the buckling of circular conical shells under uniform axial loading by setting Poisson's ratio equal to zero [5]. Two different solution methods to analyze asymmetric elastic buckling of axially compressed conical shells available in the literature were extensively compared by *Pariatmono and Chryssanthopoulos* [6]. *Tavares* expressed the mathematical approach to identify the stresses, strains, and displacements of complete or thin conical shells loaded along the meridian [7]. *Thinvongpituk and El-Sobky* examined the buckling behavior of aluminum conical shell under axial loading using the experimental setup and numerical model [8].

Differently from the current literature, this study focuses on the load carrying capacity of the conical shells with a base angle less than 25° which have flexible boundary ring under axial loading. This area has a lack of knowledge in the literature. Therefore, the main goal of the study is assigned to propose of a new method to estimate load carrying capacity of the conical shell structure with a base angle less than 25° for different radial stiffnesses under axial loading. Thus, the load carrying capacity of the conical shells which stay in the non-linear area, that is mentioned above, can be estimated without any need of numerical analysis. The study also proposes two dimensionless similarity parameters. These parameters allow for evaluation of the load carrying capacity of the conical shell for numerous configuration of geometrical dimensions in a wide range.

2 DEFINITION OF THE MODEL

The base angle is appointed as $10° \leq \propto_c \leq 20°$. According to these values, the equivalent radius of the conical shell, r_e, (it is also curvature radius at the botom of the shell, see Eq. 1) is set between 730 and $1440\,\mathrm{mm}$. The width of the circumferential ring b_{ring} is chosen as a constant value which is 15 mm. In the presented case, the stiff pipe on top of the shell also characterizes an adjacent part to simulate real condition more precisely. The height of the relatively stiff pipe h is assigned as $10\,\mathrm{mm}$. Cross section area of the circumferential ring is evaluated between $6 \leq A_{ring} \leq 300\,\mathrm{mm}^2$.

The thickness of the shell t_{shell} is set $0.5 \leq t_{shell} \leq 4\,mm$ interval. r_e/t_{shell} dimensionless parameter is assigned depending on the equivalent radius and the thickness of the shell between $240 \leq r_e/t_{shell} \leq 2880$. Additionally, the model is performed without ring (no radial stiffness) and with infinite stiff ring (fixed supported) in order to determine extremities of the limit load. The problem is simplified with a constant upper radius value as r_1=50 mm. On the other hand, the value of r_2 is selected 250 mm, initially. But, it is also used in similarity parameters as a variable. Equivalent radius of the conical shell is considered in the study as Eq. 1 [1].

$$r_e = \frac{r_2}{cos\beta_c} \quad [1]$$

3 MATERIAL AND METHOD

In this study, geometrically nonlinear analysis (GNA) is performed, and the elastic limit load is carried out. At the first step of the study, the two limit conditions, which are fixed and simple supported conical shells, are evaluated. It is important to see the extremities of the load carrying capacity. In further studies, the limit load of the conical shell for various radial stiffnesses, which is represented by a circumferential ring, is investigated. Schematic representation of the conical shell with the dimensions and the numerical model are illustrated in Figure 1 and 2. Numerical models and non-linear static analyses have been performed using the FEM program COSMOS/M with arc length control procedure. For the numerical analysis, large displacement module and Quadrilateral thick Shell element (SHELL4T) are assigned. Models are generated for three base angles α_c (10°, 15°, and 20°).

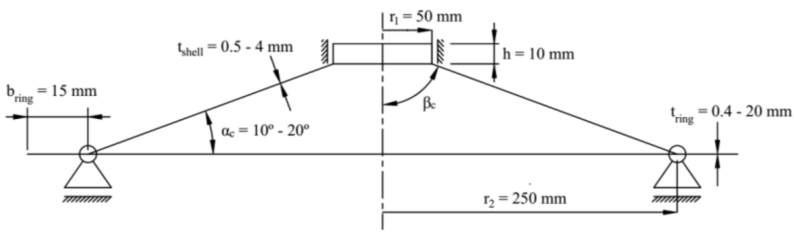


Fig. 1 Schematic representation of the conical shell with the circumferential ring.

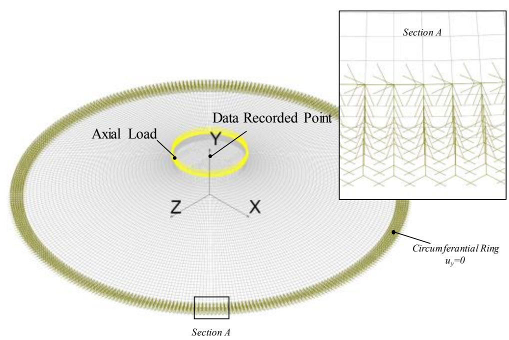


Fig. 2 Simplified numerical model.

4 RESULTS AND DISCUSSION

Figure 3 exhibits the displacement curve of the selected node number 21, which located at the top of the conical shell, depending on the axial load.

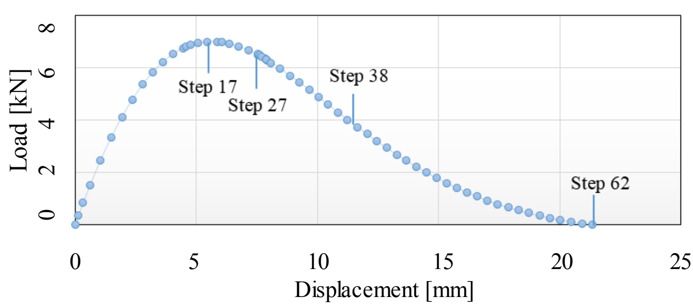
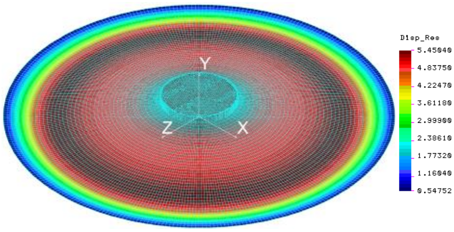
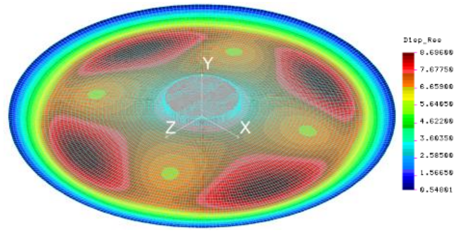


Fig. 3 Load-displacement characteristic for nodal point 21.

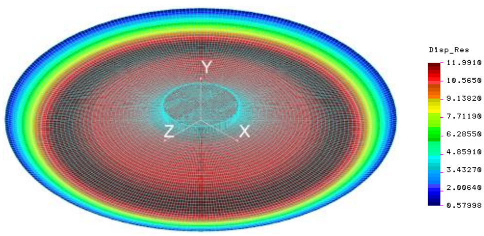
During the axial load increase, the structure reaches the limit state and loses its stability gradually. The load carrying capacity of the structure in parallel with geometrical stiffness decreases after this point.



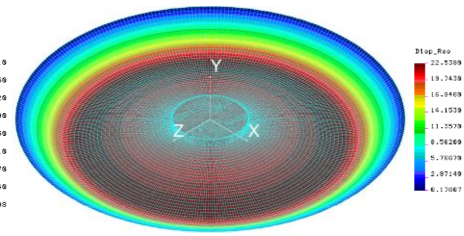
a. Resultant displacement – step number 17



b. Resultant displacement – step number 27



c. Resultant displacement – step number 38



d. Resultant displacement – step number 62

Fig. 4 Resultant displacement of the simple supported conical shell for α_c=10°, t_{shell}=1.2 mm.

The shape of the deformed structure is plotted in Figure 4. At step 17, axially symmetric deformation and nonlinear collapse occur, in this case, the top of the conical shell has 5.45 mm vertical displacement under an axial load of 6.58 kN. At step 27, the possible bifurcation point occurs and it leads to the formation of four axially symmetric waves. During the subsequent process such as step 38, the load carrying capacity of the conical shell still decreases in the post-buckling process. The structure has axially symmetric deformation. Finally, at step 62, deformation propagation needs nearly zero load value. The structure gets invert shape when it is compared to initial shape at this point.

4.1 Influence of Boundary Conditions

The limit load is substantially dependent on selected boundary conditions (Figure 5). Possible displacement at radial direction causes a reduction in load carrying capacity of the structure. The significance of the boundary condition against the load carrying capacity of the conical shell increases, especially at the lower r_e/t_{shell} values.

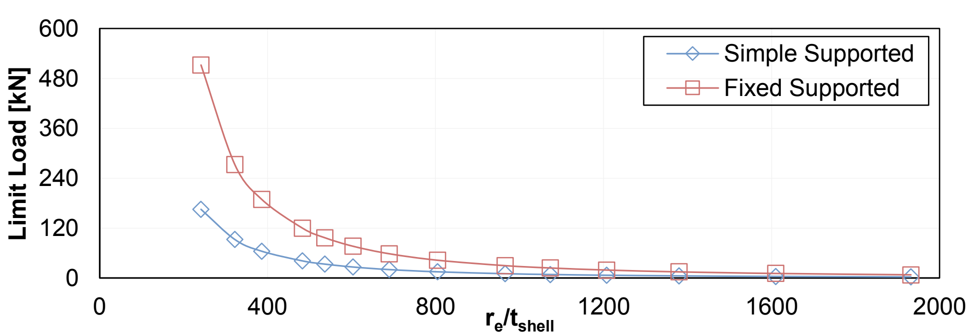


Fig. 5 Influence of the boundary conditions on the load carrying capacity for base angle 15°.

The results of the fixed supported conical shell (wholly restricted radial displacement) suggest that the circumferential ring stiffness is quite efficacious on the limit load of the structure. The relation between the limit load and r_e/t_{shell} parameter of the conical shell with base angle 15° is illustrated in Figure 5.

4.2 Influence of Base Angle

The influence of the base angle on the load carrying capacity at the various shell thicknesses is illustrated in Figure 6. From the curves, it is obviously seen that the conical shell with a higher base angle for the same shell thickness has a relatively larger amount of load carrying capacity. The strength of the structure against axial loading increases with both the shell thickness and the base angle. The limit load of the structure is nearly related to the square of the shell thickness. Data are well matched with a second order power function of the shell thickness. On the other hand, the increment of the base angle, even just one degree, gives a serious contribution to the limit load, positively. Since the increment of the base angle provides reducing the bending state effect which is caused by the nature of the structure. The proportion of the radial component of the force at the lower edge also decreases as base angle increasing.

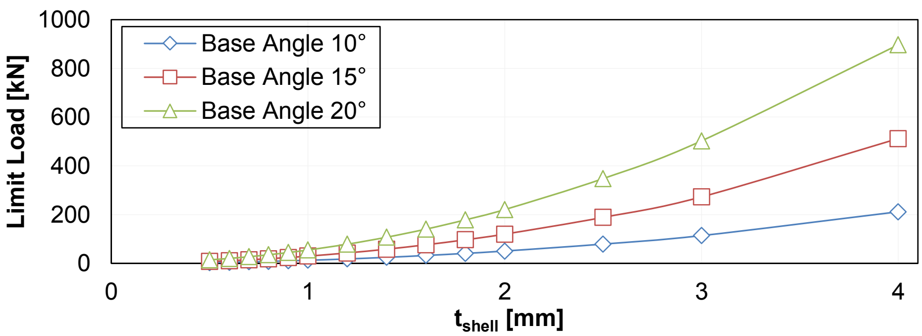


Fig. 6 Effect of the base angle on the load carrying capacity for the fixed supported conical shell.

4.3 Conical Shell with Circumferential Ring

In the previous sections, the load carrying capacity of the simple supported and fixed supported conical shells is mentioned. These boundary conditions at the lower edge are the representations of the extremities. However, in the practical applications, the conical shell is used with the boundary conditions which are located between two extremities (with a circumferential ring). This part of the study aims to derive simple relationships corresponding to geometrical parameters

The following figure is exhibited the limit load depending on r_e/t_{shell} for different circumferential ring stiffnesses. As expected, the curves which belong to various ring cross-sectional areas (different radial stiffness) are positioned between two extremities. It is interesting that the ring area even $A_{ring} = 6\ \mathrm{mm}^2$ contributes significantly positive effect to the load carrying capacity of the conical shell. On the other hand, $A_{ring} = 300\ \mathrm{mm}^2$ provides nearly same contribution with the infinite stiff case.

It is apparently seen that the importance of the radial stiffness on the conical shell structures which have a base angle less than 25°, in Figure 7. The capability of load carrying can reach three times higher in the comparison between the structures which have a cross-sectional area of the circumferential ring of $A_{ring} = 300\ \mathrm{mm}^2$ and $A_{ring} = 6\ \mathrm{mm}^2$ in the lower r_e/t_{shell} ratios. However, this difference decreases in higher r_e/t_{shell} ratios. This situation is related to the slenderness of the structure. In higher r_e/t_{shell} ratios, the expected limit load is relatively low. Therefore, the circumferential ring with $A_{ring} = 6\ \mathrm{mm}^2$ also behaves stiff enough against the radial displacement until the nonlinear collapse occurs. Hence, the limit loads of the structures with $A_{ring} = 6\ \mathrm{mm}^2$ and $A_{ring} = 300\ \mathrm{mm}^2$ become nearly same in case of quite high amount of r_e/t_{shell} values.

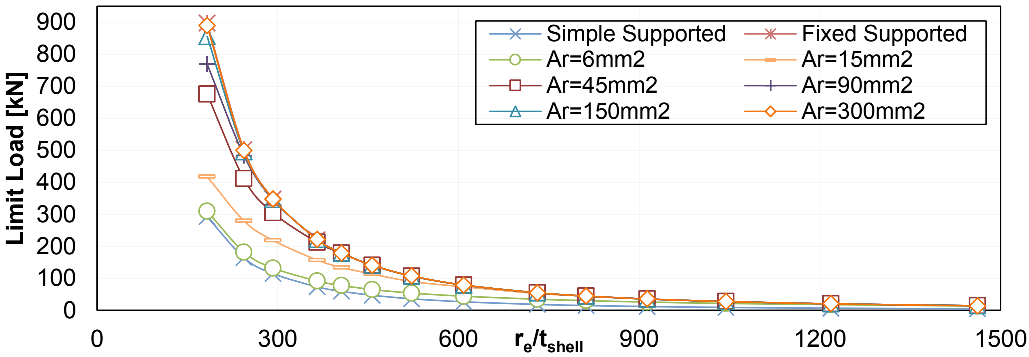


Fig. 7 Limit loads of the conical shell with different radial stiffnesses for base angle 20°.

4.4 Similarity Criteria

The load carrying capacity of the conical shell which has r_1=50 mm and r_2=250 mm was investigated up to now in the present study. But, this section mentions about the derived similarity parameter. Thus, the load carrying capacity of many different configurations of the conical shells can be estimated. For instance, a large conical shell which is used under operation can be simulated with a simple model using similarity parameters. In addition to this, the load carrying capacity of the structure can be calculated via Eq. 4 non-dimensionally without any need to a numerical analysis.

According to the results, a similarity between the load carrying capacities of the conical shells regarding geometrical parameters is tried to derive. To achieve this purpose, the load is normalized by a constitutive relation with respect to the cross-section area of the lower edge. Therefore, normalized axial load (Eq. 2) is adapted to the results as exhibited in the literature before [9]. It is a function of limit load and geometrical parameters of the structure; besides, it represents the limit load in nondimensional form.

$$F_{Normalized} = \frac{F_{lim}}{2\pi r_2 t_{shell} E} \qquad [2]$$

In this study, dimensionless Γ rigidity parameter of the circumferential ring depends on the radius of the lower edge of the conical shell, the thickness of the shell and the cross-sectional area of the ring. The parameter is represented in Eq. 3.

$$\Gamma = \frac{r_2 t_{shell}}{A_{ring}} \qquad [3]$$

The function that is seen in Eq. 4 gives results in maximum 15% variation when it is compared to FEM. The normalized load can calculate with this equation using a and b from Table 2 corresponding to base angle and rigidity parameter Γ.

$$F_{Normalized} = a\left({r_e}/{t_{shell}}\right)^{-b} \qquad [4]$$

The aforementioned non-dimensional similarity parameters are r_e/t_{shell} and Γ. If these parameters are identical for the same base angle, the normalized load of these structures is expected to be equal. The numerical analyses results and obtained values from Eq. 4 for randomly selected conical shell structures are seen in Table 1. The structures which are expected to operate in real applications have different upper and bottom radii. It is seen that Eq. 4 has good agreement with the FEM results. Besides, the similarity parameters are well matched. The structures with various geometrical dimensions but same similarity parameters have a similar normalized load. In addition to this, if the rigidity parameter of the structure is not found in Table 2, linear interpolation is used to get coefficients.

Tab. 1 FEM and analytical results for the conical shells with base angle 10°

	$r_1[mm]$	$r_2\ [mm]$	$t_{shell}\ [mm]$	r_e/t_{shell}	Γ	${F_{Normalized}}^{*}$ 10^6 [-](FEM)	${F_{Normalized}}^{*}$ 10^6 [-](Eq. 4)
A_1	100	500	5	575.88	20	83.54	85.8
A_2	250	500	5	575.88	20	82.80	85.8
A_3	300	2000	20	575.88	20	86.81	85.8
A_4	800	2000	20	575.88	20	87.53	85.8
A_5	700	5000	50	575.88	20	86.95	85.8
A_6	2000	5000	50	575.88	20	88.04	85.8

Tab. 2 Coefficients of the conical shell for parameter Γ.

Base Angle $\propto_c$ [°]	**Range of** r_e/t_{shell} **parameter**	**Rigidity Parameter** $\Gamma = \frac{r_2 t_{shell}}{A_{ring}}$	**Coefficients** a	b
10	480 - 2880	Fixed Supported	0.0696	0.995
		Simple Supported	0.0190	1.001
		1	0.1652	1.067
		5	0.1173	1.066
		10	0.0569	0.987
		20	0.0286	0.913
		40	0.0371	0.957
		60	0.0508	1.015
		80	0.0546	1.044
		100	0.0417	1.015
15	320 - 1930	Fixed Supported	0.1141	0.999
		Simple Supported	0.0289	0.998
		1	0.1697	1.032
		5	0.1320	1.028
		10	0.0814	0.979
		20	0.0424	0.899
		40	0.0515	0.948
		60	0.0700	1.008
		80	0.0730	1.032
		100	0.0614	1.025
20	240 - 1460	Fixed Supported	0.1526	0.991
		Simple Supported	0.0375	0.996
		1	0.2634	1.038
		5	0.2036	1.033
		10	0.1230	0.984
		20	0.0566	0.880
		40	0.0730	0.946
		60	0.0936	1.006
		80	0.0937	1.023
		100	0.0650	0.992

5 CONCLUSION

In this study, the load carrying capacity of the conical shell structures which have different radial stiffnesses is examined. The base angle of the conical shell structures is kept less than 25° to contribute to filling the deficiency in the literature. A new method is proposed to estimate the load carrying capacity for mentioned conical shell structures. Results which are obtained from the nonlinear FEM analyses are stated below.

In order to predict load carrying capacity of the conical shell structures under the axial load with lower base angles (i.e. 10, 15 and 20°), nondimensional design parameters (Γ and r_e/t_{shell}) are derived. Based on these parameters, a similarity approach is proposed which estimates load carrying capacity of the shells of different shell geometry configurations at the same base angle. This similarity approach tells that the two different shell configurations having the same Γ, r_e/t_{shell} and base angle have the same normalized loads. Practically, this provides an enormous advantage of estimating load carrying capacity of the conical shells from small to large structures. Therefore, there is no need to perform some series of the experiments to determine the load carrying capacity of the structures. The sensitivity on imperfection is less dominant on these type non-standard structures. However, the influence of the imperfections on the carrying capacity of the structure should be investigated in detail at further works to accomplish the research fully.

A simple expression is proposed to calculate the normalized load of the conical shell structure as a function of the dimensionless geometrical shell parameters and two constant coefficients of "a" and "b" which are selected considering the base angle, rigidity parameter. In this way, it enables an appropriate prediction of the load carrying capacity of the conical shell structures under the axial load for a variety of the shell configurations without performing some complex non-linear FEM analysis or numerical solutions. Furthermore, the discrepancy of the proposed new method and FEM results of the normalized load is found out to be the maximum 15%, which can be considered in the acceptable limits for a highly nonlinear shell behavior of the lower base angles.

Implementation of the linear theory in the load carrying capacity calculations concludes with the high amount of deviations due to the presence of the circumferential ring and highly nonlinear shell response of the shell structures, which is encountered at low base angles such as 10, 15 and 20º. The proposed expression for the normalized load minimizes this aforementioned deviation and keeps the results within the acceptable limits. Since particular equation coefficients of "a" and "b" are selected in order to characterize the non-linear response of the corresponded shell geometry.

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