

# Learning Interval-Valued Fuzzy Cognitive Maps with PSO algorithm for Abnormal Stock Return Prediction

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**Abstract.** Stock return prediction is considered a challenging task in financial domain. The existence of inherent noise and volatility in daily stock price returns requires a highly complex prediction system. Generalizations of fuzzy systems have shown promising results for this task owing to their ability to handle strong uncertainty in dynamic financial markets. Moreover, financial variables are usually in difficult to interpret causal relationships. To overcome these problems, here we propose an interval-valued fuzzy cognitive map with PSO algorithm learning. This system is suitable for modelling complex nonlinear problems through causal reasoning. As the inputs of the system, we combine causally connected financial indicators and linguistic variables extracted from management discussion in annual reports. Here we show that the proposed method is effective for predicting abnormal stock return. In addition, we demonstrate that this method outperforms fuzzy cognitive maps and adaptive neuro-fuzzy rule-based systems with PSO learning.

**Keywords:** stock market, interval-valued fuzzy cognitive map, PSO algorithm, abnormal stock return.

## 1 Introduction

Soft computing methods have recently attracted increasing attention in stock market prediction problems. This is mainly associated with the existence of inherent noise and volatility in stock market prices. On one hand, highly complex prediction systems are required to address this issue. On the other hand, these systems should not only be accurate but also easy to interpret. Developing transparent prediction models has become crucial especially after the recent financial crisis.

To capture the nonlinear behaviour of stock markets, intelligent systems such as neural networks [1], fuzzy systems [2] and evolutionary algorithms [3] have been extensively applied. Lately, hybrid systems have received increased interest because they integrate the advantages of multiple methods [4]. Thus, a high prediction accuracy can be achieved, without neglecting the interpretability of the system. Fundamental and technical financial indicators are usually used as the inputs of these systems. In addition, sentiment and other textual analyses have been rapidly developed to predict stock market movements [5,6]. It has been shown that the

combination of financial indicators and linguistic analysis of firm-related documents, such as news stories and corporate annual reports, may result in more accurate stock market predictions. This is mainly because these textual sources carry complementary information about the stock's current and future prospects, which is reflected by market participants in stock prices [7]. Thus, financial and linguistic indicators are causally related. On one hand, the linguistic indicators reflect the past financial performance. On the other hand, the linguistic indicators inform about future financial performance, affecting the expectations of market participants. Despite this empirical evidence, no one as far as we know has proposed a prediction system that incorporates these causal relationships. Here we attempt to fill this gap and propose an interval-valued fuzzy cognitive map (IVFCM) with particle swarm optimization (PSO) learning. Financial and linguistic indicators extracted from corporate annual reports are used as the inputs of this prediction system.

In traditional fuzzy cognitive maps (FCMs), causal relationships between variables are represented by directed edges labelled with fuzzy weights. Using interval-valued fuzzy sets (IVFSs) instead of fuzzy sets enables the incorporation of a higher level of uncertainty. Various learning algorithms have been proposed to automatically develop FCMs [8]. The set of variables is usually provided by an expert and the learning algorithm is then used to compute the weight matrix that best fits the data. Evolutionary approaches such as genetic algorithms (GAs) [9], particle swarm optimization (PSO) [10] and memetic algorithms (MAs) [11] have been particularly effective in learning FCMs. A comprehensive survey of nature-inspired metaheuristic algorithms was conducted in [12]. Interactive evolutionary computing has used partial expert estimations to handle incompleteness and natural uncertainty in expert evaluation [13]. Moreover, the learned FCM model was further improved by a multi-local and balanced local MA search algorithm in [14]. Chi and Liu [15] proposed a multi-objective evolutionary algorithm to address the issue of the high density of learned FCMs.

The problem of conventional FCMs in financial markets is that the precise values of a weight matrix must be determined under strong uncertainty in the dynamic environment of financial markets. This can be overcome by extending the concept of fuzzy sets in recently introduced FCM generalizations, such as IVFCMs [16], fuzzy grey cognitive maps (FGCMs) [17,18], intuitionistic FCMs [19], interval-valued intuitionistic FCMs [20] and granular FCMs [21]. These systems all enable additional freedom in assigning the membership degrees to variables and causal relationships, but they have different motivation and inference mechanisms. Here we use FCMs based on IVFSs because in financial domain intervals are used to express the uncertainty related to the context or to the lack of model accuracy [16]. In the proposed prediction system, causal relationships in IVFCMs are estimated by using PSO. PSO was selected because it was effective in the dynamic optimization of FCMs in related time series forecasting problems [22]. We demonstrate that the proposed method is effective for one-day-ahead prediction of abnormal stock return. We also show that this method outperforms conventional FCMs [10], and the generalization of an adaptive neuro-fuzzy inference system (ANFIS) [23] with PSO learning.

The remainder of this paper is organized as follows. Section 2 briefly reviews previous literature on stock market prediction using fuzzy systems and their generalizations. In Section 3, we first provide the theoretical background on IVFCMs, and then we propose an IVFCM with PSO learning. Section 4 presents the data used for abnormal stock return prediction. In Section 5, we show the experimental results and compare the prediction accuracy with several neuro-fuzzy methods, such as conventional FCMs, adaptive neuro-fuzzy inference systems and intuitionistic neuro-fuzzy networks with PSO learning. Finally, we discuss the results and conclude this paper.

## 2 Related Literature on Stock Market Prediction

Here we briefly review hybrid soft computing systems that have been applied to stock market prediction. These systems usually incorporate the uncertainty processing and interpretability of fuzzy systems and integrate it with the learning capacity of neural networks or evolutionary algorithms.

ANFIS represents a typical neuro-fuzzy rule-based architecture applied for the task of stock market prediction. ANFIS was adopted by [24] to predict stock market return on the ISE National 100 Index. It was reported that the performance of stock price prediction can be significantly improved by using ANFIS. A similar architecture of Takagi-Sugeno fuzzy rule-based system was presented by [25], where a linear combination of the significant technical index was applied as a consequent to predict the stock price. A fuzzy rule-based expert system developed by [26] for portfolio managers performed superior relative to the benchmark stock market indexes. Short-term stock trends during turbulent stock market periods were predicted by using two ANFISs, one used as the controller and the other one as the stock market process [27].

To model additional uncertainty associated with stock market environment, interval type-2 fuzzy rule-based systems have been proposed. The empirical analyses showed that type-2 fuzzy systems outperform conventional type-1 fuzzy systems in predicting stock prices. Technical and fundamental indexes were used as the input variables of the proposed type-2 fuzzy rule based expert system in [28]. Type-2 fuzzy rules were generated automatically by a self-constructing clustering method and the obtained type-2 fuzzy rules were refined by a PSO learning algorithm for TAIEX and NASDAQ stock price prediction [29]. This approach outperformed conventional regression models, neural networks, fuzzy time series, and support vector regression. An integrated functional link interval type-2 fuzzy-neural system was presented by [30] for predicting stock market indexes. The model used a Takagi-Sugano type fuzzy rule base with type-2 fuzzy sets in the antecedent parts and the outputs from the functional link artificial neural network (FLANN) in the consequent parts. The parameters of all the prediction models were optimized by PSO. This approach performed better than FLANN, type-1 fuzzy logic system and local linear wavelet neural network irrespective of the time horizons spanning from 1 day to 1 month. An evolutionary interval-valued fuzzy rule-based classification system was developed by [31] for the prediction of real-world financial applications with imbalanced data. The

proposed system outperformed C4.5 decision tree, type-1, and interval-valued fuzzy counterparts that use the synthetic minority oversampling technique.

### 3 IVFCMs with PSO Learning

#### 3.1 FCMs

To effectively model nonlinear causal relationships, an FCM combines recurrent neural networks and fuzzy logic. It can be defined as a signed fuzzy weighted digraph with  $N$  nodes, where every node represents a concept. Fuzzy value  $c_i^k$ , usually within the range of  $[0,1]$ , is assigned to the  $i$ -th concept, where  $k$  denotes the index of iteration. The sign and strength of the causal relationship from concept  $j$  to concept  $i$  is expressed by using fuzzy weight  $w_{ji}$  in the range of  $[-1,1]$ . Thus, the positive and negative relationships between the concepts can be represented. In other words, in the case of positive (negative) fuzzy weight  $w_{ji}$ , an increase in  $c_j^k$  will cause an increase (decrease) in  $c_i^{k+1}$ . When calculating the new value of the  $i$ -th concept, multiple edges connected to this concept usually have to be considered. Nonlinear activation function  $f$  (usually a sigmoid-type function) is then used to transform the linear values of the concepts. The new value of the  $i$ -th concept can be calculated as follows:

$$c_i^{k+1} = f\left(c_i^k + \sum_{\substack{j=1 \\ j \neq i}}^N c_j^k \times w_{ji}\right), \quad (1)$$

where  $i$  and  $j$  denote the  $i$ -th and  $j$ -th concepts, respectively,  $k$  is the index of iteration and  $N$  is the number of concepts in the FCM. This recurrent neural network consists of  $N$  neurons organized in one layer and connected with each other, this is with  $N \times (N-1)$  synapses.

#### 3.2 Inference in IVFCMs

To reformulate reasoning in conventional FCMs, interval-valued fuzzy sets are used instead of fuzzy sets. In interval-valued fuzzy set  $A$ , the membership degree of an element  $x \in X$  is defined by an interval as follows:

$$A = \left\{ \langle x, M_A(x) \rangle \mid x \in X \right\}, \quad (2)$$

where the interval function  $M_A: X \rightarrow D [0,1]$  such that  $x \rightarrow M_A(x) = [\mu_A^L(x), \mu_A^U(x)]$  denotes the lower and upper extremes, respectively, of the interval  $M_A(x)$ ,  $0 \leq \mu_A^L(x) \leq 1$ ,  $0 \leq \mu_A^U(x) \leq 1$ . The degree of uncertainty of  $x$  can be expressed as:

$$\pi_A(x) = \mu_A^U(x) - \mu_A^L(x) \quad (3)$$

and this represents the length of the interval  $M_A(x) = [\mu_A^L(x), \mu_A^U(x)]$ .

Generally, reasoning in IVFCMs can be expressed as [16]:

$$c_i^{k+1} = \left\{ \left[ \mu_A^L(c), \mu_A^U(c) \right]_i \right\}^{k+1} = f \left( \left\{ \left[ \mu_A^L(c), \mu_A^U(c) \right]_i \right\}^k \oplus \left( \oplus_{\substack{j=1 \\ j \neq i}} \left\{ \left[ \mu_A^L(c), \mu_A^U(c) \right]_j \right\}^k \otimes \left[ \mu_A^L(w), \mu_A^U(w) \right]_{j_i} \right) \right) \quad (4)$$

Let  $A$  and  $B$  be interval-valued fuzzy sets. The addition, subtraction and multiplication operators for  $A$  and  $B$  used in this study are based on pseudo- $t$ -representable  $t$ -norms, see [32] for more details. Specifically, the operators used in Eq. (4) can be defined as follows:

$$A \oplus B = \left\{ \left\langle x, [\min(\mu_A^L(x) + \mu_B^U(x), \mu_A^U(x) + \mu_B^L(x)), \mu_A^U(x) + \mu_B^U(x))] \right\rangle \mid x \in X \right\}, \quad (5)$$

$$A - B = \left\{ \left\langle x, [\mu_A^L(x) - \mu_B^U(x), \max(\mu_A^L(x) - \mu_B^L(x), \mu_A^U(x) - \mu_B^U(x))] \right\rangle \mid x \in X \right\}, \quad (6)$$

$$A \otimes B = \left\{ \left\langle x, [\mu_A^L(x) \cdot \mu_B^L(x), \max(\mu_A^L(x) \cdot \mu_B^U(x), \mu_A^U(x) \cdot \mu_B^L(x))] \right\rangle \mid x \in X \right\}. \quad (7)$$

### 3.3 PSO Learning of IVFCMs

Here we use PSO algorithm [33] to learn the weight matrix  $\mathbf{W}=\{w_{ji}\}$ ,  $j \neq i$ , of an IVFCM. The traditional variant of PSO was used to obtain results comparable to those produced by different neuro-fuzzy methods. PSO is a population based stochastic optimization algorithm that finds the global best solution by adjusting the trajectory (velocity and position) of individual particle towards its best location and towards the best particle of the entire population according to the following equations:

$$v_{i,j}(t+1) = \omega_i v_{i,j}(t) + c_1 r_{1,j} [p_{i,j}(t) - x_{i,j}(t)] + c_2 r_{2,j} [p_{g,j}(t) - x_{i,j}(t)], \quad (8)$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1), \quad (9)$$

where  $\omega_i$  is the inertia weight for the  $i$ -th particle,  $c_1$  and  $c_2$  are constants (cognitive and social parameter, respectively),  $r_{1,j}$  and  $r_{2,j}$  are uniformly distributed random numbers in the range  $[0,1]$ ,  $p_{i,j}(t)$  is the best position of the  $i$ -th particle remembered, and  $p_{g,j}(t)$  is the best swarm position. The particle velocity at any instant is usually limited to  $v_{\max}$ .

The weight matrix  $\mathbf{W}$  comprises  $N(N-1)$  variables, where  $N$  is the number of concepts in the IVFCM. Each particle encodes a set of  $2N(N-1)$  variables because both lower and upper bounds  $[\mu_A^L(w_{ji}), \mu_A^U(w_{ji})]$  have to be learned. In other words, each particle represents a candidate IVFCM. Root mean squared error (RMSE), used as the fitness function, was calculated as the difference between the actual abnormal stock return of training data  $y_m$ ,  $k=1, 2, \dots, M$ , and output  $\bar{y}_k$  predicted by the IVFCM. Sigmoid function was used as activation function  $f$ . To avoid overfitting, the number of iterations in the IVFCM reasoning was fixed and set to 10 [9]. To obtain defuzzified

output  $\bar{y}_k$ , we adopted the approach of [34] and calculated the average of the output interval-valued fuzzy set as  $\bar{y}_k = (\bar{y}_k^L + \bar{y}_k^U)/2$ . As a result, the fitness function can be expressed as:

$$RMSE = \sqrt{\frac{1}{M} \sum_{m=1}^M (\bar{y}_m - y_m)^2}. \quad (10)$$

The PSO learning of IVFCMs can be defined as follows:

**Algorithm 1.**

**Inputs:** training data vectors:  $\{\mathbf{c}(1), \mathbf{c}(2), \dots, \mathbf{c}(M)\}$ , number of concepts:  $N$ , population size:  $P_{size}$

**Output:** optimized weight matrix:  $\mathbf{W}$

```

For ( $i=1$  to  $P_{size}$ ) {
     $x_{velocity} \leftarrow \text{RandomVelocity}()$ 
     $x_{position} \leftarrow \text{RandomPosition}(P_{size})$ 
     $p_i \leftarrow x_{position}$ 
    If ( $\text{RMSE}(p_i) \leq \text{RMSE}(p_g)$ )
         $p_g \leftarrow p_i$ 
}
}
While (stopping condition is not satisfied) {
    For ( $x \in P$ )
         $x_{velocity} \leftarrow \text{UpdateVelocity}(x_{velocity}, p_i, p_g)$ 
         $x_{position} \leftarrow \text{UpdatePosition}(x_{position}, x_{velocity})$ 
        If ( $\text{RMSE}(x_{position}) \leq \text{RMSE}(p_i)$ )
             $p_i \leftarrow x_{position}$ 
            If ( $\text{RMSE}(p_i) \leq \text{RMSE}(p_g)$ )
                 $p_g \leftarrow p_i$ 
        }
    }
}
return  $p_g$  - particle with the best fitness value;

```

## 4 Data

In this study, we used data for 1380 U.S. firms listed on the New York Stock Exchange (NYSE) or Nasdaq. To reduce the contribution of bid/ask bounce in reaction to annual report filing (10-K filing), we followed [5] and used only the firms with a reported stock price of at least 3 USD before the filing date. To reduce the effect of risk factors for stocks, we also removed firms with market capitalisation less than 100 million USD. Publicly available EDGAR system was used as the source of corporate annual reports. All data were downloaded for the year 2013. In previous literature, the log of the market capitalisation (lnMC) was an important financial determinant of abnormal stock returns [35]. We therefore collected data for this fundamental financial indicator the Marketwatch database.

To perform textual analysis of the corporate annual reports, we first extracted the most important textual source of insider information from 10-K filings, this is management discussion and analysis section. This section provides a comprehensive overview of the firm’s business and financial condition from the management point of view. Increasing interest in the analysis of firm-related narratives can be partly attributed to the requirements of the U.S. Securities and Exchange Commission for electronic filings. To maintain the interpretability of the prediction system, we used dictionary-based approach to calculate overall sentiment in the texts. More precisely, a finance-specific dictionary developed by [5] was used to measure the sentiment. Following previous studies [6], we used the raw term frequency of positive and negative word categories. The overall sentiment was defined as the count of positive words minus the count of negative words, divided by the sum of both positive and negative word counts.

This study also used the readability of the texts as another input. Specifically, Gunning fog index was applied as the most commonly applied readability measure. The Gunning fog index can be calculated as  $0.4 \times (\text{words per sentence} + \text{percent of complex words})$ , where complex words are words with three syllables or more.

To analyse the semantic content of the texts, we first identified top 2000 terms (without stop-words) in terms of traditional term frequency inverse document frequency weighting scheme. Then, latent semantic analysis (LSA) combined with cosine similarity was performed [36]. The LSA was carried out using singular value decomposition to transform the original feature space to a low-dimensional semantic space. Documents with the same semantic concepts can then be detected. The number of concepts was 26. The weights of these concepts were then used to calculate cosine similarity to documents separately for two classes, one with positive and the other one with negative abnormal return (note that this was performed on training data only).

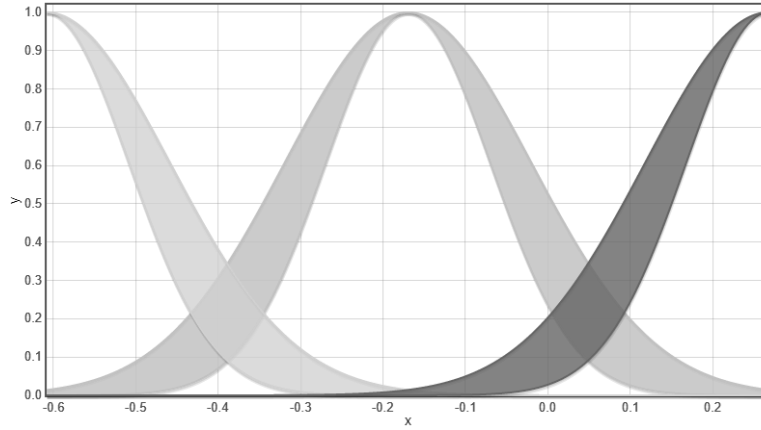
Following previous studies [5,6], abnormal returns were calculated as accumulated returns in excess of the return on the Center for Research in Security Prices equal-weighted market portfolio. In agreement with these studies, we adopted a three-day event window (prediction horizon), from day  $t-1$  to  $t+1$ , where  $t$  represents the 10-K filing day. Basic descriptive statistics of the data are presented in Table 1.

**Table 1.** Descriptive statistics of the data.

Category	Variable	Mean±St.Dev.
financial	lnMC	7.82±1.70
sentiment of annual report	Overall sentiment (Sent)	-0.34±0.10
readability of annual report	Gunning fog index (Fog)	10.53±0.82
cosine distance of concepts to stocks with positive abnormal return	LSA <sub>pos</sub>	0.161±0.109
cosine distance of concepts to stocks with negative abnormal return	LSA <sub>neg</sub>	0.168±0.105
predicted output	Abnormal stock return in $t+1$	0.004±0.072

## 5 Experimental Results

PSO algorithm was used to learn the weight matrix of the IVFCM. The setting of the PSO was as follows: population size  $P_{size}=40$ , cognitive parameter  $c_1=2$ , social parameter  $c_2=2$ , inertia weight  $\omega_i=0.8$ , maximum particle velocity  $v_{max}=0.4$ , and maximum number of function evaluations=20000 as termination criteria. To avoid overfitting, all experiments were performed by using 10-fold cross-validation (90% selected as the training set, the remaining 10% used as the testing set, repeated 10 times). Hereinafter, we report RMSE and MAPE (mean absolute percentage error) on testing data. All the values of the input variables in the IVFCM were transformed to a range of [0,1] using center of gravity defuzzification method applied to regularly distributed lower and upper Gaussian membership functions with 0.1 and 0.15 spread, respectively. For example, Fig. 1 shows these functions for the overall sentiment variable. The initial value of abnormal stock return were collected from day  $t-3$  to  $t-1$ .



**Fig. 1.** Interval-valued membership functions for overall sentiment

Table 2 presents the average values of the interval-valued fuzzy weights in the trained IVFCMs. The large differences between the lower and upper bounds of the weights denote strong uncertainty in the causal relationships between  $LSA_{neg}$  and ASR (abnormal stock return) and  $lnMC$  and ASR, respectively. For the sake of interpretability [9], the weights in the interval  $[-0.1,0.1]$  were not included in Table 2. In agreement with theoretical assumptions,  $lnMC$  had a strongly negative effect on ASR. Similarly, the effect of  $LSA_{pos}$  and  $LSA_{neg}$  on ASR was positive and negative, respectively. In contrast, the effect of readability (Fog) was rather weak. To demonstrate the increase in the uncertainty of the concept values, Fig. 2 shows the average values of the target concept ASR on the testing data. The increase corresponds to financial reality, as the degree of uncertainty is larger in the long run.

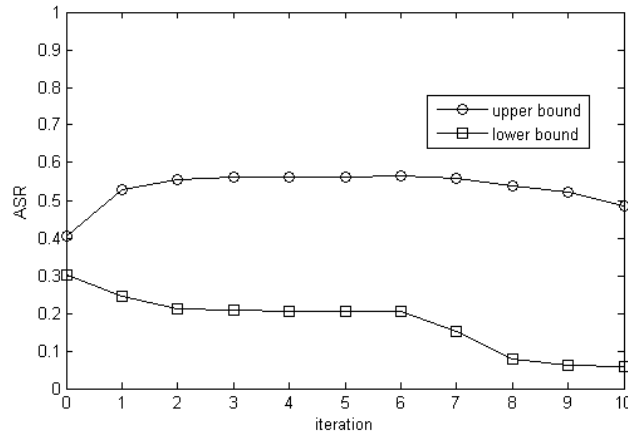
To compare the performance of the IVFCM-PSO, we employed conventional linear regression and three neuro-fuzzy methods with PSO learning: (1) FCM-PSO [10]; (2)



ANFIS-PSO [23]; and (3) intuitionistic neuro-fuzzy network (INFN-PSO) [23]. In the traditional FCM-PSO, each population comprises  $N(N-1)$  particles, thus having lower computational complexity compared with the IVFCM-PSO. In the experiments with the FCM-PSO, we used the same setting of the PSO parameters.

**Table 2.** Weight matrix of the IVFCM trained with PSO (average over 10 experiments).

	lnMC	Sent	Fog	LSA <sub>pos</sub>	LSA <sub>neg</sub>	ASR
lnMC				[-0.11,0.06]	[-0.17,0.13]	[-1.00,0.06]
Sent	[-0.12,0.01]		[-0.12,0.04]		[-0.11,0.09]	[-0.41,0.04]
Fog				[-0.14,0.08]		[-0.11,0.04]
LSA <sub>pos</sub>		[-0.15,0.09]	[-0.02,0.11]		[-0.26,0.31]	[-0.06,0.14]
LSA <sub>neg</sub>	[-0.26,0.11]	[-0.23,0.08]	[-0.26,0.12]	[-0.18,0.15]		[-1.00,-0.26]
ASR	[-0.10,0.11]	[-0.12,-0.04]			[-0.54,-0.05]	



**Fig. 2.** Average lower and upper bounds of ASR in IVFCM iterations

The ANFIS-PSO and INFN-PSO are rule-based neuro-fuzzy systems trained by using PSO. Here, we adopted the Pittsburgh approach to evolutionary-based fuzzy systems, where each particle encodes a set of  $M$  if-then rules. The PSO was used to tune the parameters of membership (and in the case of the INFN also non-membership) functions in the antecedent and parameters of the linear functions in the consequents of the if-then rules. Therefore, the dimension of each particle depends on both the length of if-then rules  $N$  and the number of if-then rules  $M$ , as each particle in the PSO comprises  $M(4N+(N+1))$  variables [23]. The learning of the ANFIS and INFN was performed in two steps. First, cluster centers (and thus also the number of membership/non-membership functions and number  $M$  of if-then rules) were found by using the subtractive clustering algorithm. To control complexity (granularity) and avoid the potential overfitting risk, we tested various numbers of membership/non-membership functions and if-then rules,  $M=\{2, 3, 5, 7, 9\}$ , for each training dataset. Thus, interpretability at the rule base and fuzzy partition levels was preserved. In the second step, the premise and consequent parameters of the ANFIS and INFN were optimized by using PSO with the following setting: population size  $P_{\text{size}}=40$ , cognitive

parameter  $c_1=2$ , social parameter  $c_2=2$ , inertia weight  $\omega_i=0.8$ , maximum particle velocity  $v_{\max}=0.4$ , and number of iterations=500 as termination criteria.

The results in Table 3 demonstrate that the IVFCM-PSO performed best with an average RMSE of 0.051. This suggests that the causal relationships among the concepts improved prediction accuracy. Moreover, the neuro-fuzzy generalizations performed better than the standard FCMs and ANFIS, respectively, indicating strong uncertainty in the causal relationships. To test the statistical differences in the RMSE performance, we conducted the nonparametric Friedman test and two post-hoc procedures (Holm and Finner). We tested the null hypothesis that all the methods have equal ranks. Although the lowest average ranking was achieved by the IVFCM-PSO, significant differences between the evaluated methods were not detected at  $p<0.05$  (the Friedman  $p$ -value was 0.207). In the post-hoc procedures, the IVFCM-PSO was used as a control method. Significant differences at  $p<0.05$  were detected between the IVFCM-PSO and other methods by using the post-hoc procedures.

**Table 3.** Results for the IVFCM-PSO vs. comparative methods (Mean $\pm$ St.Dev. over 10 experiments).

Method	Measure	Mean $\pm$ St.Dev.	Method	Measure	Mean $\pm$ St.Dev.
IVFCM-PSO	RMSE	0.051 $\pm$ 0.042	INFN-PSO	RMSE	0.055 $\pm$ 0.047
	MAPE	2.45 $\pm$ 0.45		MAPE	2.51 $\pm$ 0.42
FCM-PSO	RMSE	0.061 $\pm$ 0.045	Linear Regr.	RMSE	0.072 $\pm$ 0.054
	MAPE	2.52 $\pm$ 0.49		MAPE	2.79 $\pm$ 0.62
ANFIS-PSO	RMSE	0.059 $\pm$ 0.048			
	MAPE	2.61 $\pm$ 0.56			

## 4 Conclusion

In this study, we demonstrate that the proposed novel IVFCM with PSO learning can be effectively used to the one-day-ahead prediction of abnormal stock returns. Similar to other economic and business domains, the interpretability of the relationships in terms of IVFSs is also appropriate for predicting abnormal stock returns. In addition, the degree of uncertainty increased over time, which is in agreement with financial expectations. We also demonstrate that the reasoning based on IVFSs can be for this task more effective than that based on traditional fuzzy sets or rule-based neuro-fuzzy inference systems.

Several important limitations need to be considered regarding the present study. Alternative evolutionary approaches can be used to learn FCMs. Specifically, the variants of GAs [37] and other optimization algorithms such as enhanced PSO, MA, differential evolution and artificial bee colony can be used to learn IVFCMs. Moreover, the current study only examined the learning of an IVFCM weight matrix. This may be noted as the main weakness of this study. Further experimental investigations are therefore needed to optimize the slope parameter of each sigmoid activation function. It would also be interesting to vary the IVFCM densities by using a multi-objective evolutionary algorithm. In fact, small weights can be excluded as no real-life map considers weak relationships. Finally, more complex prediction

problems should be examined to investigate the scalability of IVFCMs with PSO learning.

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