Analysis of Short – Base Multistatic Systems Receiving Aircraft Transponders Signals

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***Abstract*— In the paper a mathematical model of a short-base multistatic system using aircraft transponder signals is presented. Several system modifications, combining a hyperbolic TDOA system, elliptical system using its own interrogator, angle measurement system and system exploiting the altitude information from the aircraft are simulated and analyzed. Their horizontal and vertical position estimation RMS errors are compared and possibilities of ADS-B based positioning system reliability enhancement are proposed.**

Keywords—Time difference of arrival (TDOA); Angle of arrival (AOA); multi-lateration (MLAT); Automatic Dependent System – Broadcast (ADS-B); Air traffic control (ATC); Air traffic management (ATM);

I. Introduction

Permanent air traffic density growth calls inevitably for continuous development of further aircraft position and movement control tools enhancing tracking accuracy and reliability and expanding system communication capability. In this situation the recent spread of the ADS-B (Automatic Dependent Surveillance – Broadcast) [1] technology brought a significant progress in this issues. In the USA and EU projects of new air traffic control and management (ATC/ATM) technologies development based mainly on this principle (i.e. the NextGen in USA and the SESAR in EU) are under way ([2], [3], [4]). Some countries (i. e. Australia, India) even now leave the existing ATC techniques, based on secondary surveillance radars adopting only the ADS-B based surveillance technology.

This technology undoubtedly offers a perfect survey of ADS-B equipped aircraft positions with high accuracy based on satellite navigation systems services. But any reliable ATC system needs to use independent data of more than of a single source to eliminate its errors and drop-outs due to source failures or jamming or its incorrectness due to intentional actions.

Time difference of arrival (TDOA) based multilateration systems are well known alternatives to secondary radars and also to the ADS-B based surveillance systems. Lately, series of authors published their analyzes of various passive multiposition systems combining TDOA, angle of arrival (AOA), received signal Doppler frequency analysis a.s.o. (i.e. [5]).

In this paper we describe an analysis of several modifications of a short-base terrestrial multiposition system supporting and enhancing accuracy and reliability of aircraft position determination using signals of secondary surveillance or ADS-B transponders at 1 090 MHz emitted from the tracked objects. The aim of the research, conducted in the Faculty of Electrical Engineering and Informatics of the University of Pardubice in 2016 is to compare various system modifications and design an effective and reliable supplement to the systems dependent solely on the ADS-B messages.

In this paper the models of the system and its modifications are described in section II. Then, in the next part the simulation results of individual system modifications are compared and evaluated.

II. Positioning System Description

All the multisite positioning system configurations studied in the following analysis are compared according to their mean square errors (RMS) of the horizontal and vertical aircraft position estimation (σ_H and σ_V). As a basis for comparison of all modifications in this paper we take the TDOA system with four ground receiving stations deployed at a constant height at vertices of an equilateral triangle and at its center. In this analysis we assume that each station receives signals from all directions.

Besides this pure TDOA system we will analyze the following system (measurement) combinations:

- TDOA system with angle measurement (AOA) capability in azimuth and elevation at all stations or at only a selected one
- Time of arrival measurement system with its own interrogator forming a combined hyperbolic and elliptical ([6], [7]) time measurement system and AOA measurement
- The system with all previous capabilities supplemented with information on the aircraft altitude using aircraft altimeter data from the transponder message [8].

It is shown, that angles measurements are beneficiary only at very short ranges. On the contrary adding an own system interrogator and including elliptical system position calculation using independent signals significantly improve the position accuracy.

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A. System parameters

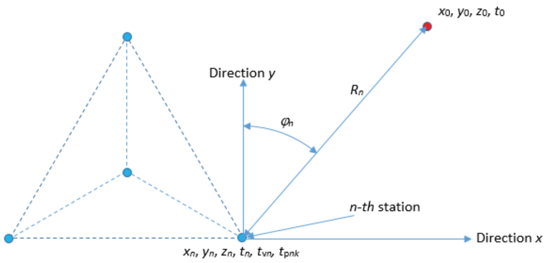

Fig. 1. Situation.

Positions of the aircraft and stations are expressed in the Cartesian coordinate system centered on the place of the bottom left station and with the xy plane tangential to the Earth's surface at this point. We use the following symbols:

x_0, y_0, z_0	location of the monitored object (a transmitter on the aircraft board):
x_i, y_i, z_i	receiving stations locations $i = 1$ to 4
t_0	the moment of spontaneous broadcasting of the on-board transponder
t_i	the moment of the spontaneous signal arrrival on the i-th station, $i = 1$ to 4
T_0	the moment of the interrogation broadcasting
T_i	the moment of the response arrival at the i-th station
h	aircraft altitude measured by an on-board altimeter

B. Mathematic Model

We start from a system of nonlinear equations expressing measured time instants of arrival t_i (TOA) of spontaneously broadcasted signal (hyperbolic or TDOA subsystem), times of arrival of the aircraft response on the interrogation of its own interrogator T_i (elliptical subsystem), the measured values of target azimuth φ_i and elevation θ_i at individual stations and a given flight altitude h:

$$t_n = t_0 + \frac{R_n}{c} + \delta t_n \quad (1)$$

$$R_n = \sqrt{(x_0 - x_n)^2 + (y_0 - y_n)^2 + (z_0 - z_n)^2} \quad (2)$$

$$\begin{aligned} &\sin(\varphi_n) = \frac{(x_0 - x_n)}{R_{0n}} + \delta\{\sin(\varphi_n)\} \\ &for\ \varphi_n \in \langle -45°, 45°\rangle \cup \langle 135°, 225°\rangle \\ &\cos(\varphi_n) = \frac{(y_0 - y_n)}{R_{0n}} + \delta\{\cos(\varphi_n)\} \\ &for\ \varphi_n \in \langle 45°, 135°\rangle \cup \langle 225°, 315°\rangle \\ &\sin(\theta_n) = \frac{(z_0 - z_n)}{R_n} + \delta\{\sin(\theta_n)\} \end{aligned} \quad (3)$$

$$T_n = T_0 + T_{odp} + \frac{R_K + R_n}{c} + \delta T_0 + \delta T_{odp} + \delta T_n \quad (4)$$

$$h = h_0 + \delta h; \quad h_0 \cong \frac{x_0^2 + y_0^2}{2R_Z} + z_0 \quad (5)$$

Where:

δt_n	is the measurement error of the TOA of a spontaneously transmitted signal at the n-th station
$\delta\{\ldots\}$	is a measurement error of the variable in the brackets
δT_0, δT_{odp}, δT_n	are measurement errors of interrogation broadcasting time, of the on board response time delay and of TOA of the response at the n-th station
δh	is the error of the aircraft altitude determination
R_Z	is the Earth radius
K	is the numerical order of the station with the interrogator.

After linearization of the system of equations it is possible to calculate the scattering matrix of the aircraft position estimation using the equation:

$$\mathbf{S} = \mathbf{var}[\delta\mathbf{P}] = (\mathbf{C}^{\mathbf{H}}. \mathbf{W}^{-1}.\mathbf{C})^{-1} \quad (6)$$

where

C is a matrix of the linearized system:

$$\mathbf{C} = \begin{Vmatrix} \mathbf{C1} \\ \mathbf{C2} \\ \mathbf{C3} \\ \mathbf{C4} \\ \mathbf{C5} \end{Vmatrix}; \quad \begin{aligned} \mathbf{C1} &= \|C1_{nk}\| \\ \mathbf{C2} &= \|C2_{nk}\| \\ \mathbf{C3} &= \|C3_{nk}\| \\ \mathbf{C4} &= \|C4_{nk}\| \\ \mathbf{C5} &= \|C5_{1k}\| \end{aligned} \quad (7)$$

where **C1** to **C4** are 4x4 matrices and **C5** is a 1x4 vector. The matrices **C1**, **C2**, **C3** correspond to the hyperbolic TDOA measurement subsystem and to azimuth and elevation measurements respectively and their expressions could be found in [5].

$$C1_{n1}=\frac{\partial R_n}{\partial x_0};C1_{n2}=\frac{\partial R_n}{\partial y_0};\ C1_{n3}=\frac{\partial R_n}{\partial z_0};\ C1_{n4}=1;$$

$$\begin{aligned}
&C2_{n1}=\frac{R_{0n}-(x_0-x_n)\frac{\partial R_{0n}}{\partial x_0}}{R_{0n}^2};\quad C2_{n2}=-\frac{(x_0-x_n)\frac{\partial R_{0n}}{\partial y_0}}{R_{0n}^2}\\
&for\varphi_n\in\left(-45°,45°\right\rangle\cup\left(135°,225°\right\rangle\\
&C2_{n1}=\frac{-(y_0-y_n)\frac{\partial R_{0n}}{\partial x_0}}{R_{0n}^2};\quad C2_{n2}=\frac{R_{0n}-(y_0-y_n)\frac{\partial R_{0n}}{\partial y_0}}{R_{0n}^2};\\
&for\ \varphi_n\in\left(45°,135°\right\rangle\cup\left(225°,315°\right\rangle\\
&C2_{n3}=C2_{n3}=0;
\end{aligned}\tag{8}$$

$$\begin{aligned}
&C3_{n1}=-\frac{(z_0-z_n)\frac{\partial R_n}{\partial x_0}}{R_n^2};\quad C3_{n2}=-\frac{(z_0-z_n)\frac{\partial R_n}{\partial y_0}}{R_n^2};\\
&C3_{n3}=\frac{R_n-(z_0-z_n)\frac{\partial R_n}{\partial z_0}}{R_n^2}
\end{aligned}\tag{9}$$

The matrix **C4** corresponds to the elliptical measurement subsystem using its own interrogator and the **C5** to the measurement of the aircraft altitude. The corresponding expressions are shown here:

$$\begin{aligned}
&C4_{n1}=\frac{\partial R_1}{\partial x_0}+\frac{\partial R_n}{\partial x_0};C4_{n2}=\frac{\partial R_K}{\partial y_0}+\frac{\partial R_n}{\partial y_0};\\
&C4_{n3}=\frac{\partial R_1}{\partial z_0}+\frac{\partial R_n}{\partial z_0};C4_{n4}=0;\\
&C5_1=\frac{x_0}{R_z};C5_2=\frac{y_0}{R_z};C5_3=1;C5_4=0;
\end{aligned}\tag{10}$$

W is the covariance matrix of measurement errors:

$$\mathbf{W}=\begin{Vmatrix}\mathbf{W}_1&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}_2\\\mathbf{0}&\mathbf{W}_2&\mathbf{0}&\mathbf{0}&\mathbf{0}_2\\\mathbf{0}&\mathbf{0}&\mathbf{W}_3&\mathbf{0}&\mathbf{0}_2\\\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{W}_4&\mathbf{0}_2\\\mathbf{0}_3&\mathbf{0}_3&\mathbf{0}_3&\mathbf{0}_3&\mathbf{W}_5\end{Vmatrix};\tag{11}$$

Where **0**, $\mathbf{0_2}$, $\mathbf{0_3}$ are zero-valued matrices and:

$$\mathbf{W}_1=(c\sigma_\tau)^2\begin{Vmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{Vmatrix};$$

$$\begin{aligned}
&\mathbf{W}_2=\sigma_\varphi^2\begin{Vmatrix}W_2(1)&0&0&0\\0&W_2(2)&0&0\\0&0&W_2(3)&0\\0&0&0&W_2(4)\end{Vmatrix};\\
&W_2(k)=\cos^2(\varphi_k);\\
&\varphi_n\in\left(-45°,45°\right\rangle\cup\left(135°,225°\right\rangle\\
&W_2(k)=\sin^2(\varphi_k);\\
&\varphi_n\in\left(45°,135°\right\rangle\cup\left(225°,315°\right\rangle
\end{aligned}\tag{12}$$

$$\mathbf{W}_3=\sigma_\theta^2\begin{Vmatrix}\cos^2\theta_1&0&0&0\\0&\cos^2\theta_2&0&0\\0&0&\cos^2\theta_3&0\\0&0&0&\cos^2\theta_4\end{Vmatrix};\tag{13}$$

$$\mathbf{W}_4=(c\sigma_{T0K})^2\begin{Vmatrix}1&0.95&0.95&0.95\\0.95&1&0.95&0.95\\0.95&0.95&1&0.95\\0.95&0.95&0.95&1\end{Vmatrix};\tag{14}$$

$$\mathbf{W}_5\equiv\sigma_h^2$$

where:

σ_τ is a RMS error of TOA measurement
σ_φ is a RMS error of the azimuth measurement
σ_θ is a RMS error of the elevation measurement
σ_{T0K} is a total RMS error of the response TOA
σ_h is a RMS error of the aircraft altitude measurement

The total RMS error of the response TOA comprises the error of the interrogation transmission instant determination, the error of the response delay on the board and the TOA measurement error at the receiver station. Due to a high share of the response delay error generated on the board and common to all stations on the total RMS error of the response TOA, measurements at individual receiving stations are highly correlated (see eq. 13 for $\mathbf{W_4}$).

Further comment: In the presented relations for the correlation matrix **W** a reception of uncorrelated signals used in hyperbolic and elliptical subsystems are presumed. In a real situation it is more likely that the stations will have the use of only the transponder response at a given time window. Then a full correlation of signals TOA used in the both subsystems take place and the system should be reduced. It was found, that dropping three equations of the elliptical subsystem (i.e. measurements at three stations) and simultaneously measurement at the complementary station in the hyperbolic subsystem leads only to a minor deterioration of the system parameters even in the case of uncorrelated signals. That is why in the following analysis we use equations (7) - (13) with these restrictions.

C. Parameters of Simulation

On the basis of this mathematical description simulations were executed with the following parameters:

- $\sigma_\tau = 3$ ns
- $\sigma_\varphi = 0.5°$
- $\sigma_\theta = 0.5°$
- $\sigma_{T0K} = 30$ ns
- $\sigma_h = 90$ m
- The station forms a triangle with the bottom left peak in the coordinate system center. The stations are located at the vertices of a triangle, which lies on circle with a radius of approximately 577 m.

III. Simulation Results

In the following figures Fig. 2. a-d results of simulations are shown for the target altitude of 100 m in a square area of ±20 km. Here we can see the horizontal and vertical RMS errors of aircraft position estimation. In the figure a. and b. errors of the pure hyperbolic system are presented, fig. c. and d. show an improved system with measurement of azimuth and elevation at the central station. From the figure 2c we can see that the azimuth measurement has only insignificant effect. On the other hand, the elevation measurement (fig. 2d) substantially refines the aircraft altitude estimation.

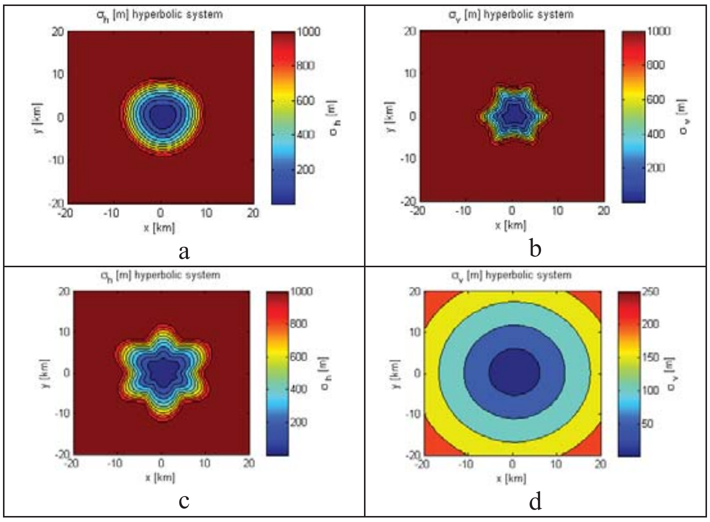

Fig. 2. Simulation in a square area of 20 x 20 km, the target altitude 100 m.

Fig 3. a-f show a situation in the 10 km flight level and in the extended area to ±350 km. A hyperbolic subsystem supplemented with azimuth and elevation measurement at the central station presented in the fig. 3a and b. is effective only in the range of several kilometers from the center station. Adding an elliptical system the horizontal errors decrease down to 450 meters at the range of 350 km (fig. 3c.) and also the vertical error (fig. 3d.) is partially improved.

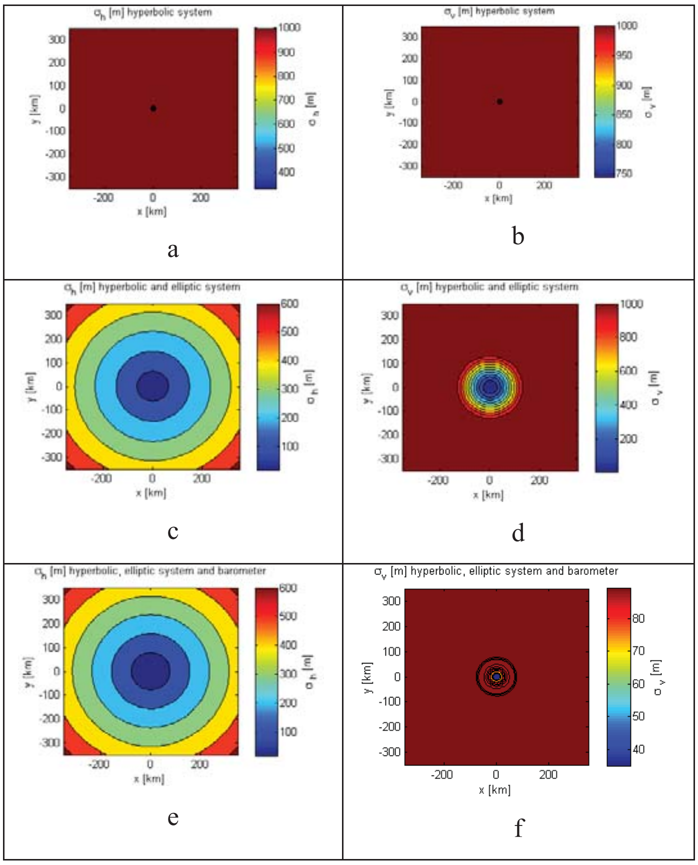

Fig. 3. Simulations in a square area of 700 x 700 km, the aircraft altitude 10 km.

Utilizing also information of aircraft barometer altitude measurement (fig. 3f.) the vertical estimation error could be suppressed down to the barometer height measurement error, which is approximately 90 m (see the colorbar).

But it should be noted, that the system interrogator will most likely use a broad-beam antenna, resulting in substantial maximum range reduction. For instance if the interrogator azimuth beam width is about 30° the maximum interrogation range would be of 150 km only.

IV. Conclusion

We analyzed several configurations of a multisite short-base system using signals of airborne transponders at 1 060 MHz for aircraft position determination. It was shown, that adding an interrogator the system horizontal error could be suppressed substantially even at a long range. Improvement of the vertical aircraft position estimation was achieved using information on the on-board measured altitude. On the other hand a system supplementation with angle measurement has an observable impact on vertical position only at short ranges.

Results of this research conducted at the Faculty of Electrical Engineering of the University of Pardubice during 2016 will serve for the design of a system enhancing accuracy and reliability of a short basis multi-lateration (MLAT) system receiving SSR aircraft transponder signals.

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