

SCIENTIFIC PAPERS  
OF THE UNIVERSITY OF PARDUBICE  
Series B  
The Jan Perner Transport Faculty  
1 (1995)

**THE STATISTICAL ACCEPTANCE OF THE STEEL BRIDGE  
STRUCTURE MEMBERS**

Hynek ŠERTLER <sup>a)</sup>, Josef VIČAN <sup>a)</sup>, Jiří SLAVÍK <sup>b)</sup>

<sup>a)</sup> Department of Infrastructure, University of Pardubice  
<sup>b)</sup> Department of Transport Linies, UTC in Žilina

**1. Introduction**

The proposed paper indicates the possibility of the application of statistical methods on the acceptance of steel bridge members. The research results of the reliability of the compression members at the Department of Civil Structures and Bridges at UTC in Žilina are there reflected [3].

**2. Resistance of the compression members**

The resistance of material or of the structure members against the load actions is a random variable composed of entrance factors of the reliability reserve  $G$  given by the formula

$$G = R - S \quad (1)$$

The value of the resistance  $R$  determined by certain small probability of the occurrence of unfavourable state presents the ultimate limit strength of the structure or its member.

For a designer, this value presents the design resistance  $R_d$ . The best way for the determination of the member resistance  $R_d$  is testing. Generally, testing is very expensive and therefore not desirable. In the most cases, the calculation is used to determine  $R_d$ . The creation of a real mathematical model is most important for the calculation. The determination of  $R_d$  for tension and single compression members is in [1], the practical utilisation of this check procedure is in [2] and [3]. For the compression members it is necessary to use such a procedure that takes into account the influence of the member buckling. The main factors of this influence are the slenderness ratio of the member, the cross-section shape and the geometrical and structural imperfections. The influence of these random variables for the reliability of the compression members is analysed in [2] and [3].

The resistance of the compression members could be expressed by the formula

$$R_c = \chi \cdot f_y \cdot \varphi_a \quad (2)$$

where

$f_y$  is the real value of the yield stress

$\varphi_a = \frac{A}{A_n}$  is the ratio of the real and nominal area of the member cross-section,

$\chi$  is the factor which includes the influences of the random variable parameters at the resistance from the view point of local and global stability failure.

For struts or columns  $\chi$  represents the reduction factor for the relevant buckling mode, for plates, it is the buckling factor.

The knowledge of the real value of the buckling resistance determines the quality of the investigated sample of the products. This fact predicts the reliability investigation for the statistical acceptance procedure of the steel members.

It is only necessary to demonstrate that the probability of the occurrence of real resistance of the set of members  $R_c \leq R_{d,c}$  is smaller than beforehand fixed failure probability  $P_f$ . This fact is expressed by the inequality

$$P(R_c \leq R_{d,c}) \leq P_f \quad (3)$$

where

$R_{d,c}$  is the standard value of the compression member strength.

If the condition (3) is satisfied, the whole set of tested members can be taken as satisfactory from the point of view of their resistance against the loading actions.

In the opposite case, the critical value of at least one entrance parameter in relation (2) can be fixed to satisfy the relation (3). From the point of view of the statistical acceptance of steel members, only those products can be accepted which have the value of imperfection smaller than their critical value.

### 3. Statistical acceptance of the set of compression struts

If the strut calculation model as a uniaxial compression member with the initial bow in the sinusoidal shape is accepted, the relation (2) could be written in the following form

$$R_c = \left\{ \frac{f_y + \left(1 + \frac{e_0}{r}\right) \cdot \sigma_{Cr}}{2} - \sqrt{\frac{\left(f_y + \left(1 + \frac{e_0}{r}\right) \cdot \sigma_{Cr}\right)^2}{4} - f_y \cdot \sigma_{Cr}} \right\} \cdot \varphi_a, \quad (4)$$

where

$e_0$  is the amplitude of the sinusoidal initial bow of a strut,

$r$  is the radius of gyration of effective strut cross-section,

$\sigma_{Cr} = \frac{\pi^2 E}{\lambda^2}$  is the elastic critical buckling stress,

$\lambda$  is the slenderness ratio,

$E$  is modulus of elasticity.

If the initial bow amplitude is expressed in the form according to [4]

$$e_0 = r \cdot \kappa \cdot \left(\frac{\lambda}{\lambda_y}\right)^2, \quad (5)$$

the relation for  $\kappa$  could be derived in the form

$$\kappa = 8820 (e_0 / L) (z_j / L) (235 / f_y). \quad (6)$$

When the relation (6) is included to (4) with using the assumption that the radius of gyration could be expressed as a linear function of the depth of cross-section and taking into account the relations

$$\varphi_L = (L / L_a), \quad \varphi_h = (h / h_n), \quad \varphi_e = (e_0 / L) / (e_0 / L)_n = (\bar{e}_0 / \bar{e}_{0n}), \quad (7)$$

$$\bar{z}_{jn} = z_{jn} / L_n, \quad (j = y, z),$$

the resulting relation for the buckling stress is given by the formula

$$R_{y,z} = \left\{ \begin{array}{l} 0,5 \left[ f_y + \pi^2 E \left( \left( \frac{\varphi_h}{\varphi_L \lambda_n} \right)^2 + \varphi_c e_{0n} \overline{z_{jn}} \frac{\varphi_h}{\varphi_L} \right) \right] - \\ - 0,25 \sqrt{\left[ f_y + \pi^2 E \left( \left( \frac{\varphi_h}{\varphi_L \lambda_n} \right)^2 + \varphi_c e_{0n} \overline{z_{jn}} \frac{\varphi_h}{\varphi_L} \right) \right]^2 - \frac{\pi^2 E \left( \frac{\varphi_h}{\varphi_L} \right)^2}{\lambda_n^2}} \end{array} \right\} \cdot \varphi_a \quad (8)$$

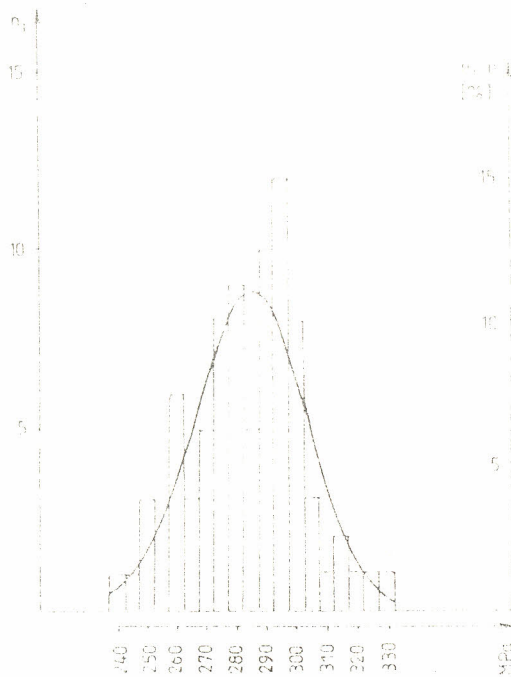
In the relations (6), (7), (8) the following denotation is used:

$L$  ( $L_n$ ) is the real (nominal) strut length,

$h$  ( $h_n$ ) is the real (nominal) cross-section depth,

$z_j$  ( $z_{jn}$ ) is the real (nominal) distance of the extreme fibres from centroid of effective strut cross-section,

$j = y, z$ .



**Fig. 1:** The histogram and probability density function of the yield stress.

Relation (8) expresses the resistance of the compression strut as a function of random variables in the form of the cross-section characteristics, slenderness ratio and the shape and amplitude of the initial bow. Using this relation, the real resistance of the set of the truss bridge structure diagonals has been analysed. The

shape  $L$  for the diagonal cross-section has been used, steel 11373. The histograms of the random variables  $f_y$ ,  $\varphi_h$ ,  $\varphi_e$  are in Fig. 1÷3, the remaining random variables have the following characteristics:

$\varphi_a$  - normal distribution with parameters

$$\mu_a = 1,05, \sigma_a = 0,065,$$

$\varphi_L$  - normal distribution with parameters

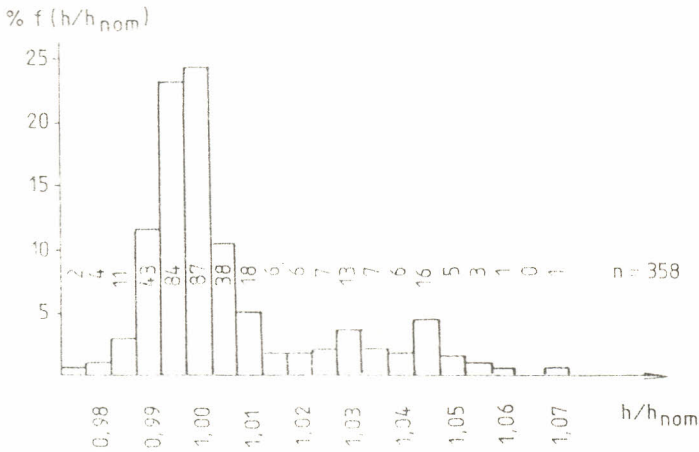
$$\mu_L = 1,0, \sigma_L = 0,00025,$$

where  $\mu_a$ ,  $\mu_L$  are the mean values of the random variables  $\varphi_a$ ,  $\varphi_L$ ,  $\sigma_a$ ,  $\sigma_L$  are their standard deviations.

Constants in the relation (8) are

$E = 210\,000$  MPa,  $\overline{z_{vm}} = 0,0101$ ,  $\lambda_n = 102,04$  according to [6],  $\overline{e_{om}} = 0,001$  according to [7],

$R_{dc} = \chi_{B} \cdot R_d = 0,53 \cdot 210 = 111,3$  MPa according to [4].



**Fig. 2:** The histogram of the random variable  $\varphi_h$ .

The Monte-Carlo technique was used, generating 2000 values of random variable  $R_C$  according to (8). The resistance of the set of compression diagonals in the value of  $R_C = 134,5$  MPa was found which corresponds to the failure probability  $P_f = 10^{-3}$ .

This value is greater than the standard value  $R_{dc} = 111,5$  MPa and the whole set of products could be accepted.

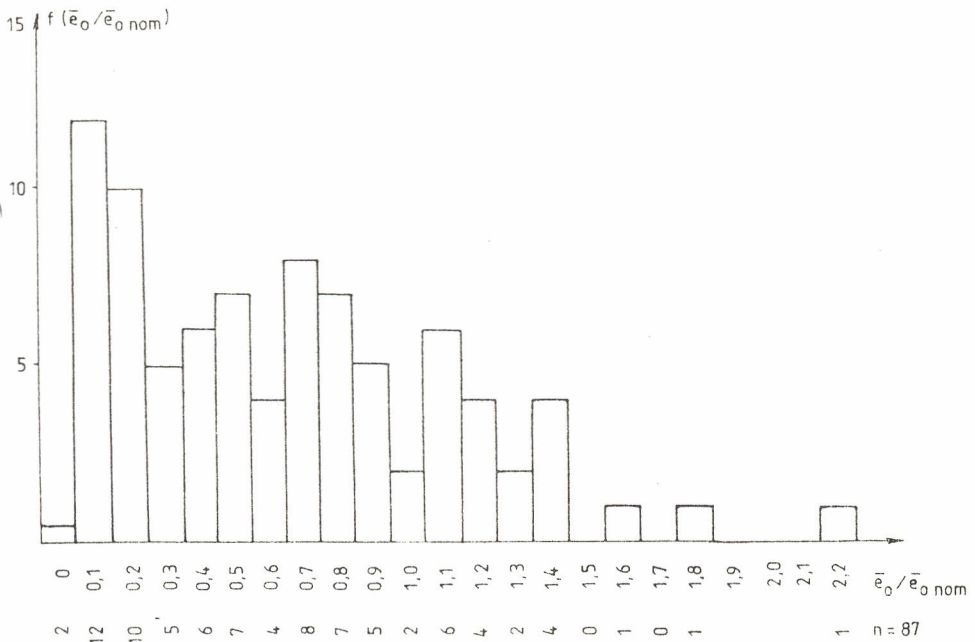


Fig. 3: The histogram of the random variable  $\varphi_e$ .

#### 4. Statistical acceptance of the compression plates

The general relation (2) for the compression plates is better to modify into the form

$$R_c = \rho_n \cdot f_y \cdot \varphi_a, \quad (9)$$

where

$\rho_n = \sigma_n / f_y$  is the real value of the buckling factor of the isotropic plate ( $\rho_n \leq 1.0$ ),

$$\rho_n = \rho_n(k_{wo}, b/t, f_y, \sigma_r), \quad (10)$$

$\sigma_n$  is the limit average stress with respect to the plate buckling.

The buckling factor  $\rho_n$  is a function of several random variables, mainly of imperfection factor  $k_{wo}$ , slenderness of plate  $b/t$ , yield stress  $f_y$  and residual stress  $\sigma_r$ . Considering all values in (10) as random variables leads to very complicated relations. As a first step the assumption that only  $k_{wo}$  is random variable, was accepted. This variable can be expressed by the formula

$$k_{w0} = 200 \cdot \Delta w_o \cdot \frac{\sqrt{245/f_y}}{b}, \quad (11)$$

where  $\Delta w_o$  is the main component of the isotropic plate buckling which depends on the measurement methods and on the evaluation of the results of measurement [7].

The dependence of  $\rho_n$  on the imperfection factor  $k_w$  is shown in Fig. 4 for sterderness of plate  $b/t = 40$ , yield stress  $f_y = 240$  MPa and 3 levels of residual stresses  $\sigma_r$  (see Fig.4).

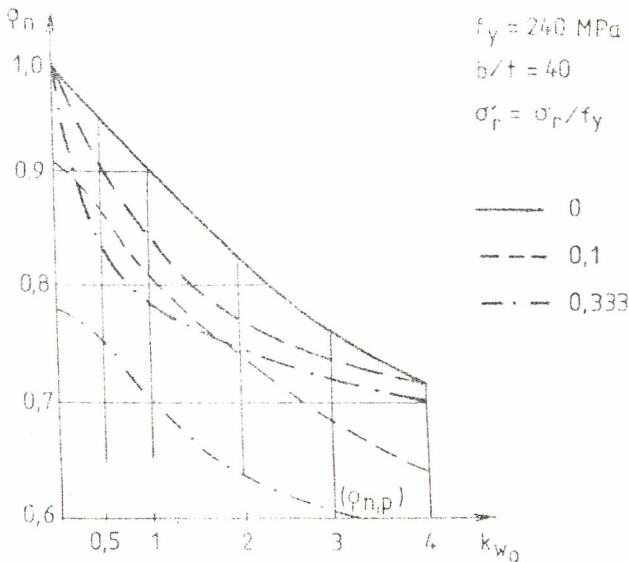
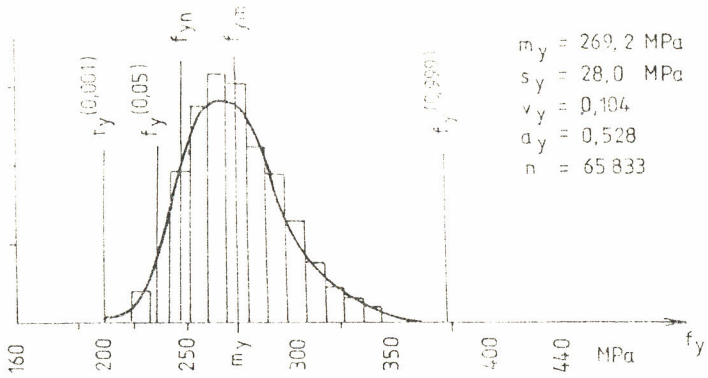


Fig. 4: The dependence of the buckling factor  $\rho_n$  on the imperfection factor  $k_{w0}$ .

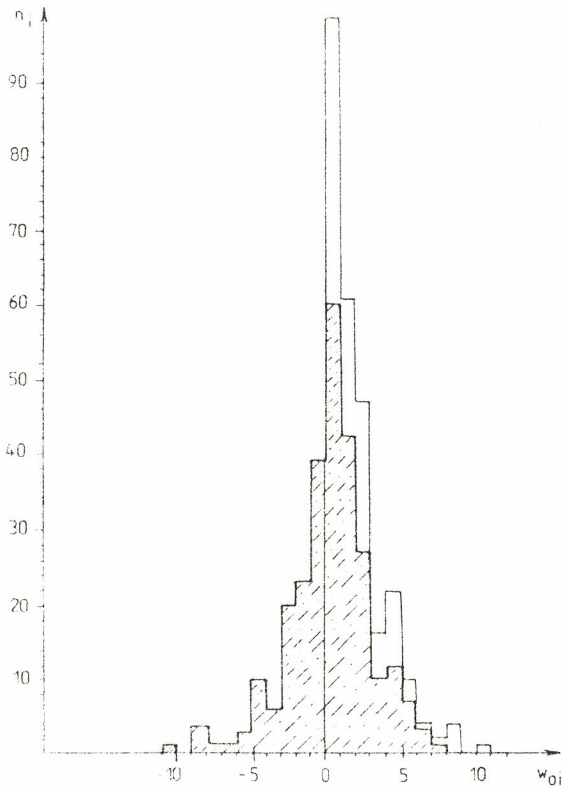
Using the plot in Fig.4 the histogram of the buckling factor  $\rho_n$  can be determined. Then applying the Monte Carlo method according to (9) the random values of the resistance of plates can be generated.

The quality of the set of products from the view point of the statistical acceptance is again determined by the relation (3).

The described method has been verified by testing isotropic plates with  $b/t = 40$  fabricated from the steel 11375 which belongs to the orthotropic plate of a road box girder. The longitudinal stiffeners are perfectly rigid in the sense of [5]. The histogram of real measured values of initial deflection amplitude according to (11) is shown in Fig. 6. The probability density function of the yield stress is shown in Fig. 5.



**Fig. 5:** The histogram and probability density function of the yield stress of isotropic plates.



**Fig. 6:** The histogram of real measured values of initial deflection amplitude of isotropic plates.



The parameters of  $\varphi_a$  are:  $\mu_a = 1,0$  ,  $\sigma_a = 0,03$  for normally distributed random variable  $\varphi_a$  .

The design resistance value of the isotropic plate according to [5] is  $R_{dc} = 0,85 \cdot 210 = 178,5$  MPa.

Using Fig. 4 for  $\sigma_a = 0,1 f_y$ , the histogram for  $\rho_n$  has been developed and the resistance of the investigated set of plates has been determined. For the failure probability  $P_f = 10^{-3}$  the resistance  $R_C = 160,7$  MPa has been found. It means that the condition (3) was not satisfied. The condition holds for  $P_f = 2 \cdot 10^{-2}$ . The value of  $P_f = 10^{-3}$  could be reached only by excluding the plate sheets with initial deflections  $k_{w0} > 1,4$ , it is for  $w_0 > 4$  mm.

## 5. Conclusions

Two examples were shown to demonstrate the method of the statistical acceptance of compression members from the view point of their reliability.

The realisation of the statistical acceptance by this method enables to determine the failure probability of the resistance of the set of products with high accuracy.

*Lektoroval: Prof. Ing. Pavel Novák, DrSc.*

Předloženo v únoru 1995.

## References

- [1] MRÁZIK, A.: Reliability theory of steel structures. (In Slovak), VEDA Bratislava 1987.
- [2] ŠERTLER, H. - VIČAN, J. - SLAVÍK, J.: Calculation of resistance failure probability of compression members of steel structures. Building Research Journal 40, Number 12/1992, p. 729-745.
- [3] ŠERTLER, H. - VIČAN, J. - SLAVÍK, J.: Utilization of statistical methods for the quality check of steel structures (In Czech). Final report of HČ 10/SvF/91 for Vítkovice's iron and steel works.
- [4] ČSN 73 1401 Design of steel structures (in Czech) ÚNM Praha 1984.
- [5] ČSN 73 6205 Design of steel bridge structures (In Czech) ÚNM Praha 1984.
- [6] MELCHER, J.: Real behaviour of built-up compression struts. Final report of research P-12-124-003-02/3c, Faculty of Civil Engineering, Brno, 1975.
- [7] ŠERTLER, H. + kol.: Dimension and shape deviations of steel structures. (In Czech) Final report of research 99/194-88/1228, UTC in Žilina, 1989.

## Summary

### THE STATISTICAL ACCEPTANCE OF THE STEEL BRIDGE STRUCTURE MEMBERS

Hynek ŠERTLER, Josef VIČAN, Jiří SLAVÍK

The paper presents a new statistical approach to the acceptance of steel bridge structure members based on their reliability. Two examples are shown to demonstrate this methods for the acceptance of compression members.

The described methods enables to determine the failure probability of the set of members with higher accuracy.

## Zusammenfassung

### DIE STATISTISCHE ÜBERNAHME DER STAHLBRÜCKENELEMENTEN

Hynek ŠERTLER, Josef VIČAN, Jiří SLAVÍK

Der Beitrag befaßt sich mit dem neuen statistischen Zutritt für die Übernahme der Stahlbrückenelementen, der an ihrer Verlässlichkeit gegründet ist. Zwei Beispiele sind angeführt, um die Methode für die Übernahme der druckbelasteten Elementen vorzustellen. Diese Methode vermägt die Wahrscheinlichkeitsverletzung der Tragfähigkeit der Stahlelementen mit höher Genauigkeit bestimmen.

## Resumé

### STATISTICKÁ PŘEJÍMKA PRVKŮ OCELOVÝCH MOSTNÍCH KONSTRUKCÍ

Hynek ŠERTLER, Josef VIČAN, Jiří SLAVÍK

Článek prezentuje nový statistický přístup k problematice přebírání prvků ocelových mostů vycházející z jejich spolehlivosti. Jsou uvedeny dva příklady, které demonstrují uvedenou metodu při přejímce tlačných prvků. Popsaná metoda umožňuje stanovit pravděpodobnost poruchy souboru zkoumaných prvků s větší přesností.