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ANALYTICAL PREDICTION OF BIFURCATION PROCESS IN AN ADHESIVE ROLLING KINEMATIC PAIR

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1. Introduction

For technical applications with the presence of passive resistance in bonds, there is generally accepted the dry fiction model and solutions for steady states of oscillation are interpreted in the form of elementary periodic functions. The development of the theory of non-autonomous systems has positively proved that even in the case of existence of actuating effect described by the elementary harmonic function there occur states of motion, which can be characterized as quasi-periodic states. If we accept the Coulomb dynamic friction model, where the friction coefficient **f** is a function of the relative speed of the contacting surfaces, we get to the field of nonlinear mechanics, where periodic steady states in the state space form isolated closed trajectories with limiting cycles (orbits), see [1].

It is necessary to point out that analysis of the discussed problem is only possible at simple dynamic models of mechanical oscillation. Its usability consists in the transparency of the analysis of processes, the existence of which is not commonly perceived, but it can have substantial influence on the erosion of contact surfaces of the adhesive rolling kinematic pair. The analysis of limiting cycles (Poincaré transformation) enables the study of so-called bifurcations (branching) of steady states of mechanical oscillation. Prediction in this case means to create a time prognosis of possible changes of limiting oscillatory motions of the nonlinear system and configuration of processes, which can be brought even to chaotic states by possible occurrence of bifurcations [2].

The behaviours of nonlinear dynamic systems strongly depends on so-called control parameters of the system. These appear in the coefficients of differential equations of the selected model. If the analysis shall have specific meaning, it is necessary to watch the model with clear physical interpretation. This paper, as its name implies, presents the model solution which should describe oscillatory motions of particles of contacting surfaces of the rolling kinematic pair of the adhesion drive of a rail vehicle.

The basic nonlinearity in this case is caused by the dependence of the Coulomb friction coefficient on the relative speed of contacting particles, which is a result of existing slips. The non-autonomousness of the selected model consists in periodical loading of any particle of the contact surface at the travel of the rail vehicle (see Chapter 3).

2. Notes on Bifurcation of Periodic Steady States

Let the system be represented by the limiting cycle described by trajectory γ in the phase plane (***Fig. 1***). Let us plot perpendicular plane **H** through any point of this trajectory and picture possible bifurcations (branching) of motions in the plane. Intersections of trajectories $\gamma_{0,1,2...n}$ with plane **H** (so-called hypersurfaces) can be considered sources of bifurcation processes [3]. Fig. 1 (according to [2]) highlights stable nodes with one parameter **p**. At specific application (see Par. 3) it is necessary to determine the mentioned intersections, which is directly related to determination of the parameters of hypersurface **H**. This is a substantial problem, because the bifurcation process occurs in such a state of the system, when the system has the possibility to choose how its motion will further develop in time. For completeness we state the mathematically important statement that the dimension of bifurcation problems depends on the number of the own numbers of the linearization matrix (marked with the **A** symbol).

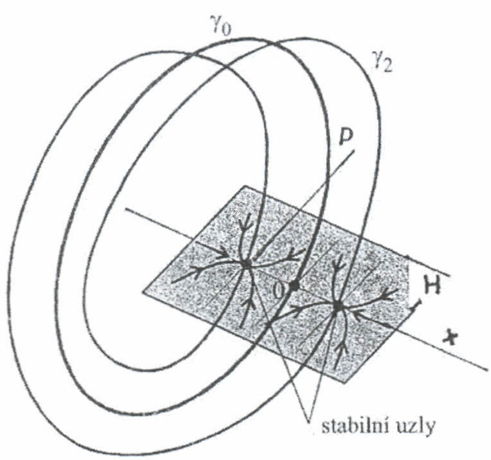

Fig. 1 *Trajectory of the limiting cycle*

The concept of this matrix directly relates to the theory of stability of non-autonomous nonlinear systems. In parallel with the term bifurcation it is necessary to understand the term **attractor** of the dynamic system, which represents the state, which the system is logically heading towards.

Limiting cycles (orbits) are related with periodic attractor. Point attractor represents the tendency of the system to achieve the steady state. Under certain conditions they may convert to so-called "strange attractors", which signal occurrence of chaotic motions.

3. Example of Analytical Determination of Bifurcation Parameter

In the sense of the application mentioned in Par. 1 we are giving the solution and analysis of motion equations of the dynamic model according to ***Fig. 2***. The weight of particle **2** of contact surface of the adhesive kinematic pair is **M.** The flexible bond of the particle with the neighbourhood **1** is represented by linear spring with stiffness **k.** Rotation of the radially-loaded kinematic pair is modelled by the actuating force **$F_1 \sin \Omega t$** acting on **2.** Its attenuation is expressed by the parallelly connected particle **3** of the weight **m** via the friction attenuator, whose instant force **F** corresponds to the relative speed of particles **2** and **3**, i.e. **F = $\varphi(\dot{v} - \dot{u})$**. (Absolute speeds $\dot{v}$ and $\dot{u}$ relate to particles **1** and **2**). The instant value of the dry friction coefficient is **f**.

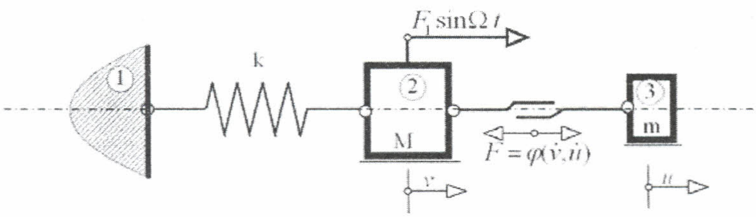

Fig. 2 *Dynamic model and its dynamic equation.*

If we introduce dimensionless quantities $\tau = \Omega t$; $\xi = \frac{M\Omega^2}{F_1}u$; $\eta = \frac{M\Omega^2}{F_1}v$; $\mu = \frac{m}{M}$;

$\omega^2 = \frac{k}{M\Omega^2}$; $\beta = \frac{F}{F_1}\frac{1}{f(\dot{\eta} - \dot{\xi})}$

the two-weight model according to Fig. 2 is described by motion equations (1)

$$\ddot{\xi} = \omega^2 \xi = \sin \tau + \beta f(\dot{\eta} - \dot{\xi}) \quad \mu\ddot{\eta} = -\beta f(\dot{\eta} - \dot{\xi}) \tag{1}$$

If we integrate equations (1) using the oscillation symmetry conditions

$$\dot{\xi}(\tau_0 + \pi) = -\dot{\xi}(\tau_0)$$
$$\dot{\eta}(\tau_0 + \pi) = -\dot{\eta}(\tau_0)$$

we get the function $y(\tau) = \dot{\eta}(\tau) - \dot{\xi}(\tau)$ representing the dimensionless relative speed of particles **2** and **3** :

$$y(\tau)=\beta\left[\frac{1}{\omega}tg\frac{\omega\pi}{2}\cos\left(\tau-\tau_0\right)-\frac{1}{\omega}\sin\omega\left(\tau-\tau_0\right)+\frac{\pi}{2\mu}\right]-\frac{\cos\tau}{\omega^2-1} \tag{2}$$

The first stage we will find from the condition **y (τ_0) = 0** :

$$\beta\left(\frac{1}{\omega}tg\frac{\omega\pi}{2}+\frac{\pi}{2\mu}\right)-\frac{\cos\tau_0}{\omega^2-1}=0 \tag{3}$$

Gradual differentiating of equation (2) by the dimensionless time **τ** will get us to first four derivations (4, 5, 6, 7). These relations (for $\tau = \tau_0$) are not written here for the sake of briefness of the text.

By using of the first derivation (4) under the condition $\tau = \tau_0$ together with the second derivation (6) we get the equation of the bifurcation plane **H_1** in the dimensionless form (8):

$$\beta^2\left(\omega^2-1\right)^2\left[\left(\frac{1+\mu}{\mu}\right)^2+\left(\frac{1}{\omega}tg\frac{\omega\pi}{2}+\frac{\pi}{2\mu}\right)^2\right]=1 \tag{8}$$

The bifurcation plane, i.e. also the arisen process of branching of the oscillation motion, disappears. If we zeroize the relations (3, 4, 5) at simultaneous elimination of time τ_0, we get the equations of lines representing intersections of plane H_1 with other two planes H_2, H_3. These are sets of subsequent possible branching of oscillation motions. The respective equations shall we get from the zeroized relations (3, 4, 5) at elimination of time **τ_0** :

$$\mu=\frac{\pi\omega}{2\left(\omega^2-1\right)}\left(tg\frac{\omega\pi}{2}\right)^{-1}$$

$$\frac{1}{\beta}=\left(\omega^2-1\right)^2\left\{\omega^2tg^2\frac{\omega\pi}{2}+\left[1+\frac{2\left(\omega^2-1\right)}{\omega\pi}tg\frac{\omega\pi}{2}\right]^2\right\} \tag{9}$$

Solution of equations (9) with the zeroized form of derivation (6) will bring the auxiliary condition (10):

$$\mu\left(\omega^2-1\right)=1 \tag{10}$$

The last two conditions determine the values of dimensionless parameters ω, μ, β, the keeping of which ensures elimination of occurrence of more complex bifurcation processes of the handled model of non-autonomous system. It is obvious that a steady state, in which steady simple harmonic motion of the handled model occurs, represents a dynamic situation, which can be easily changed towards more complex motions.

In more complicated models than the handled one there may possibly arise conditions of deterministic chaos with all possible negative consequences on the variety and degree of the erosion of contact surfaces of the monitored adhesive kinematic pair. As the analyzed rolling kinematic pair forms the basic element of drives of rail vehicles, it is necessary to take into account that complex mechanical oscillation of superficial particles of material in the contact area may significantly affect the quality of adhesive transmission of tangential (tensile) force.

That is why the authors of this paper consider it necessary to study the problems of bifurcation processes further, especially in experimental way. One of experimental laboratory options is prepared in the labs of the Jan Perner Transport Faculty, where an unique equipment for the study of adhesion characteristics in relation to torsional plasticity of the driving system was developed within the project GAČR, reg. No. 101/07/0727.

4. Conclusion

This paper presents analytical study on ambiguity of oscillatory motions of a non-autonomous mechanical system. The Coulomb model is interpreted in dependence of the dry fiction coefficient on the relative speed of the contacting surfaces, which makes the system nonlinear. There is pointed out possible occurrence of bifurcation splitting and its possible effects on the erosion of the contact rolling kinematic pair or on possible irregularities of adhesion transmission of the torque of driving rail vehicles.

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References

1. BREPTA, R., PŮST, L., TUREK, F. *Mechanické kmitání.* Technický průvodce 71, SABOTÁLES, Praha 1994.
2. JANÍČEK, P. *Systémové pojetí vybraných oborů pro techniky - hledání souvislostí.* CERM, VUTIUM, Brno 2007, Učební texty 2.
3. SAWARAGI, Y., FUJTI, T. *Forced vibrations of the system with two degrees of freedom with Coulomb damping.* Bulletin of I.S.M.E, 1959, II, No. 4.

Resumé

ANALYTICKÁ PREDIKCE BIFURKAČNÍHO PROCESU UVNITŘ ADHEZNÍ VALIVÉ KINEMATICKÉ DVOJICE

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V příspěvku je podána analytická studie o mnohoznačnosti kmitavých pohybů neautonomní mechanické soustavy. Coulombovský model je pojat v závislosti součinitele suchého tření na relativní rychlosti stýkajících se ploch, čímž soustava má nelineární charakter. Je upozorněno na možný vznik bifurkačního štěpení a jeho možných důsledků na porušování kontaktní valivé kinematické dvojice, event. na možné nepravidelnosti adhezního přenosu kroutícího momentu hnacích kolejových vozidel.

Summary

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This paper presents analytical study on ambiguity of oscillatory motions of a non-autonomous mechanical system. The Coulomb model is interpreted in dependence of the dry fiction coefficient on the relative speed of the contacting surfaces, which makes the system nonlinear. There is pointed out possible occurrence of bifurcation splitting and its possible effects on the erosion of the contact rolling kinematic pair or on possible irregularities of adhesion transmission of the torque of driving rail vehicles.

Zusammenfassung

ANALYTISCHE PREDIKTION DES BIFURKATIONSPROZESSES IM INNEREN DES ADHÄSIVEN KINEMATISCHEN WÄLZPAARES

Rudolf KALOČ, Libor BENEŠ

Im Beitrag wird eine analytische Studie betreffs der Mehrdeutigkeit der schwingenden Bewegungen des nicht autonomen mechanischen Systems geliefert. Das Coulombsche Modell ist in der Abhängigkeit des Faktors der trockenen Reibung von der relativen Geschwindigkeit der sich berührenden Flächen gefasst, wodurch das System einen nichtlinearen Charakter hat. Es wird auf die mögliche Entstehung der Bifurkationsspaltung und deren mögliche Auswirkungen auf die Störung des kinematischen Kontakt-Wälzpaares, beziehungsweise auf die möglichen Unregelmäßigkeiten der adhäsiven Übertragung des Drehmomentes der Schienen-Triebfahrzeuge hingewiesen.