Contributing to the aggregated expression of environmental damage by air pollution

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Abstract **-** This article describes one of the possible approaches to the calculation of the pollution and its summary indicator. But, the suggested total indicator of pollution needs a correction according to the bias. This correction is described in this paper on chosen example.

The aim is to separate the total pollution into aggregates indicator of material and aggregates indicator of indicator of pollution per unit of quantity of emissions.

An indicator of pollution is possible to separate into two different aspects. It is possible to define pollution indices and the indices for quantity. The article presents the process and outcome on real data of air pollution in the Czech Republic.

Keywords **-** Environmental damage, level of air pollution, price level, total value of pollution.

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I. THEORETICAL APPROACH

A. Value, Quantity, Unit Value

Let us consider a complex (total) phenomenon or process which consists of n partial disjoint parts i, i= 1, 2, …, n. The unification of all parts gives the total. [3],[12]. E.g. aggregate pollution consists of pollution caused by CO_2, SO_2, etc.[7], [8], [1].

Let us consider value of individual h_i for any part i. Let us assume that variables h_i are addable i.e. they have the same meaning and it is possible to sum them. We will call them values.

We state

$$H = \sum_{i=1}^{n} h_i \tag{1}$$

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Let us define variables $q_{i,}$ which correspond quantity (amount) of an item i. The meaning of these variables can be various. So, their sum may not make sense. These variables will be called quantity.

So, for any item i we have two variables: value and quantity.

For any item i we define

$$p_i = \frac{h_i}{q_i}. \tag{2}$$

The variable p_i means the value of item i per unit of quantity i. If the value is expressed in monetary units, these variables represent prices. Generally, it may not be so, therefore, we will use term unit value.

For any h_i and positive q_i we obtain

$$h_i = \frac{h_i}{q_i}\, q_i \tag{3}$$

$$h_i = p_i\ q_i. \tag{4}$$

Values h_i , i=1,2, …, n form vector h.

Values q_i, i=1,2, …, n form vector q.

Values p_i, i=1,2, …, n form vector p.

Coordinates of these vectors correspond to the item i, i=1,2, …, n. The sum H is the total value in chosen time t or time period (t-1,t>

$$H = \sum_{i=1}^{n} h_i = \sum_{i=1}^{n} p_i\, q_i\ . \tag{5}$$

A question arises whether it is possible to define ~~set~~ numbers P and Q which correspond to vectors p and q so that

$$H = P\,Q\,. \tag{6}$$

P and Q represent scalar representatives of vectors p and q.

B. Data Changes

In this paragraph we will consider continuous time and infinitesimal time interval < t, t + dt).

From relations written above it follows

$$dH = \sum_{i=1}^{n} dh_i = \sum_{i=1}^{n} \frac{dh_i}{h_i} h_i = \sum_{i=1}^{n} d(p_i\, q_i) = \sum_{i=1}^{n} (q_i\, dp_i + p_i\, dq_i) \quad (7)$$

$$\frac{dH}{H} = \sum_{i=1}^{n} \frac{dh_i}{H} = \sum_{i=1}^{n} \frac{dh_i}{h_i} \frac{h_i}{H} = \sum_{i=1}^{n} \frac{dh_i}{h_i} w_i \quad (8)$$

$$\frac{dH}{H} = = \sum_{i=1}^{n} \frac{d(p_i, q_i)}{p_i\ q_i} w_i = \sum_{i=1}^{n} \frac{q_i\, dp_i + p_i\, dq_i}{p_i\ q_i} w_i = \sum_{i=1}^{n} \left(\frac{dp_i}{p_i} + \frac{dq_i}{q_i}\right) w_i\ , \quad (9)$$

where weight w_i fulfils

$$w_i = \frac{h_i}{H}. \quad (10)$$

From it follows

$$\mathrm{d} \ln H = \sum_{i=1}^{n} (\mathrm{d} \ln p_i + \mathrm{d} \ln q_i)\ \ w_i \quad (11)$$

This relative change of the total value is equal to weighted sum of relative changes of the values for individual i.

Therefore, infinitesimal growth of value is equal to sum of weighted infinitesimal changes of unit value and quantity [6],[3].

In order to express infinitesimal growth we assume dependency of the variables on parameter (e.g. time) t which will be assigned as an index to considered variables. Therefore, we can write

$$H_{t+dt} = H_t + dH_t \quad (12)$$

$$\frac{H_{t+dt}}{H_t} = 1 + \frac{dH_t}{H_t} \quad (13)$$

It arises from here that the index reduced by a unit represents relative change of variable H.

$$\ln \frac{H_{t+dt}}{H_t} = \ln H_{t+dt} - \ln H_t = \sum_{i=1}^{n} w_i (\ln q_{it+dt} - \ln q_{it}) + \sum_{i=1}^{n} w_i\ (\ln p_{it+dt} - \ln p_{it}) \quad (14)$$

We assume that weights $w_i\ i = 1, 2, \ldots, n$ do not depend on parameter t.

From which arises

$$\ln \frac{H_{t+dt}}{H_t} = \sum_{i=1}^{n} w_i \left(\ln \frac{q_{it+dt}}{q_{it}} + \ln \frac{p_{it+dt}}{p_{it}}\right) = \sum_{i=1}^{n} w_i\ \ln \frac{q_{it+dt}}{q_{it}} \frac{p_{it+dt}}{p_{it}} \quad (15)$$

$$\ln \frac{H_{t+dt}}{H_t} = \sum_{i=1}^{n} \ln \left(\frac{q_{it+dt}}{q_{it}}\right)^{w_i} \left(\frac{p_{it+dt}}{p_{it}}\right)^{w_i} \quad (16)$$

$$\ln \frac{H_{t+dt}}{H_t} = \ln \prod_{i=1}^{n} \left(\frac{q_{it+dt}}{q_{it}}\right)^{w_i} \left(\frac{p_{it+dt}}{p_{it}}\right)^{w_i} \quad (17)$$

$$\frac{H_{t+dt}}{H_t} = \prod_{i=1}^{n} \left(\frac{q_{it+dt}}{q_{it}}\right)^{w_i} \left(\frac{p_{it+dt}}{p_{it}}\right)^{w_i} \quad (18)$$

Let us define

$$Q_t = K_Q\, q_{1t}^{w_1}\ q_{2t}^{w_2}\ \ldots\ q_{nt}^{w_n} \quad (19)$$

$$P_t = K_P\, p_{1t}^{w_1}\ p_{2t}^{w_2}\ \ldots\ p_{n}^{w_n}\ , \quad (20)$$

where K_Q and K_P are positive constant such that

$$K_Q\ K_P = 1. \quad (21)$$

So, we can see

$$\frac{H_{t+dt}}{H_t} = \frac{Q_{t+dt}}{Q_t} \frac{P_{t+dt}}{P_t}. \quad (22)$$

For time t;

P_t may be considered as aggregate variable of unit values, i.e. as level of unit value;

Q_t may be considered as aggregate variable of value.

II. EXAMPLE

A. Polution and its Clasification

Environmental data are available in the various classifications of pollution and other harmful effects [8], [11].

For the purposes of interpretation, it is sufficient to consider the information relating to one year and the types of pollution, which are in the following table.

Tab. 1 Selected air pollutants

(n)	Type of pollution	
1	PM_{10}	Particulate Matters
2	SO_2	Sulphur dioxide
3	NO_x	Oxides of nitrogen
4	CO_2	Carbon dioxide

Source [5]

We will use the selected data for the year 2011 in further analysis [5].

In general, you can assume n types of pollution.

B. Quantity of Polution

Default data are considered types of pollution emissions in tons per year (t/year) that are listed in the following table

Tab. 2 Emitted amount per year

(n)	q	kt/year
1	PM_{10}	48,4207345
2	SO_2	170,180470
3	NO_x	225,308640
4	CO_2	107 991,12

Sources: [10], authors, [11]

Costs of pollution per unit of quantity.

The amount of emissions we will mark the kind of q_i. These quantities are not possible to sum.

The next table shows data about the cost, which the issuer must incur pollution. Costs relating to the nature of the pollution and the unit of quantity (1 ton) we mark p_i. Their size is EUR per ton (€/t).

Tab. 3 Estimating of damage for pollutants unit

(n)	p	€/t
1	PM_{10}	11 000
2	SO_2	4 000
3	NO_x	4 000
4	CO_2	19 000

Source:[2]

The value of the pollution of the kind i we denoted by h_i. We get this value by using the relation

$$h_i = p_i \ q_i. \tag{23}$$

Dimension values h_i we determine the size of the values of p_i q_i. Applies

$$[€/Year] = [€/t] [t/ Year]]. \tag{24}$$

So, the values h_i are possible to sum for i = 1,2, ..., N.

Tab. 4 Damage for year in €

(n)	p * q	€/year
1	PM_{10}	532 628 079,5
2	SO_2	680 721 880,0
3	NO_x	901 234 560,0
4	CO_2	2 051 831 280 000,0
Sum		2 053 945 864 519,5

It is possible to define nonnegative weights prom the formulae

$$w_i = \frac{h_i}{H} \qquad i = 1,2,3,4 \tag{25}$$

So, we receive the table

Tab. 5 Weights calculation

n	Item	$h_i=p_i*q_i$ (€/Year)	w_i
1	PM_{10}	532 628 079,5	0,000259319
2	SO_2	680 721 880,0	0,000331422
3	NO_x	901 234 560,0	0,000438782
4	CO_2	2 051 831 280 000,0	0,998970477
Sum		2 053 945 864 519,5	1,000000

From relations

$$P = p_1^{w_1} \ p_2^{w_2} \ ... \ p_n^{w_{nt}} \tag{26}$$

$$Q = q_1^{w_1} \ q_2^{w_2} \ ... \ q_n^{w_n} \tag{27}$$

We determine the value of P and Q and their product.

In the calculation we use logarithms. Indeed

$$\ln(P) = \ln(p_1^{w_1}) + \ln(p_2^{w_2}) + \cdots + \ln(p_n^{w_n}) \tag{28}$$

$$\ln(P) = w_1 \ln(p_1) + w_2 \ln(p_2) + \cdots + w_n \ln(p_n) \tag{29}$$

$$\ln(Q) = \ln(q_1^{w_1}) + \ln(q_2^{w_2}) + \cdots + \ln(q_n^{w_n}) \tag{30}$$

$$\ln(Q) = w_1 \ln(q_1) + w_2 \ln(q_2) + \cdots + w_n \ln(q_n)$$

(31)

We come out of table by the logarithms of the values

Tab. 6 Logarithms of the values

n	Item	$h_i=p_i*q_i$ (€/Year)	w_i
1	PM_{10}	532 628 079,5	0,000259319
2	SO_2	680 721 880,0	0,000331422
3	NO_x	901 234 560,0	0,000438782
4	CO_2	2 051 831 280 000,0	0,998970477
Σ		2 053 945 864 519,5	1,000000

Continue of Tab. 6

n	Item	$\ln(h_i)$	$w_i \ln(h_i)$	$w_i \ln(h_i)$ and Σ
1	PM_{10}	20,0933340	0,005210592	0,005210592
2	SO_2	20,3386644	0,006740671	0,006740671
3	NO_x	20,6192761	0,009047368	0,009047368
4	CO_2	28,3497538	28,32056709	28,32056709
Σ		28,3507839	28,35078387	28,34156573

Consider the aggregate numbers

Tab. 7 Aggregate numbers

n	It corresponds to		Exp(Σ)
1	$\Sigma_i w_i = 1$	28,3507839	2 053 945 864 520
2	$\Sigma_i w_i \ln(h_i)$	28,3415657	2 035 099 290 843

Let's calculate the values of P and Q. We proceed from tables

C. The values and weights

Tab. 8 Given values and weights

n	Item	p_i (€/t)	q_i (t/year)	$h_i=p_i*q_i$ (€/year)	w_i
1	PM_{10}	11 000	48 420,7345	532 628 079,5	0,000259319
2	SO_2	4 000	170 180,4700	680 721 880,0	0,000331422
3	NO_x	4 000	225 308,6400	901 234 560,0	0,000438782
4	CO_2	19 000	107 991 120,00	2 051 831 280 000,0	0,998970477
Σ		x	x	2 053 945 864 519,5	1,000000

Tab. 9 Logarithm of values

n	Item	$\ln(p_i)$	$\ln(q_i)$	$\ln(h_i)=$ $=\ln(p_i)+\ln(q_i)$
1	PM_{10}	9,30565055	10,78768	20,09333395
2	SO_2	8,29404964	12,04461	20,33866438
3	NO_x	8,29404964	12,32523	20,61927611
4	CO_2	9,85219426	18,49756	28,34975382

Tab. 10 Logarithm of the value multiplied by weights and the sum of its

n	Item	$w_i \ln(p_i)$	$w_i \ln(q_i)$	$w_i \ln(h_i)$ $w_i (\ln(p_i)+\ln(q_i))$
1	PM_{10}	0,002413	0,002797	0,005211
2	SO_2	0,002749	0,003992	0,006741
3	NOx	0,003639	0,005408	0,009047
4	CO_2	9,842051	18,478516	28,320567
Σ		9,850852	18,490713	28,341566
EXP(Σ)		18 975	107 254 308	2 035 099 290 843

Easy to see that it is

2 035 099 290 843 = 18 975*107 254 308

$$P\ Q = {}_wH$$

(32)

So,

value of **18 975** can express scalar values representative of p_i, which denoted P

value of **107 254 308** can express scalar values representative of Q_i, which we denote Q;

value **2 035 099 290 843** can express scalar representative h_i, which denote ${}_wH$

At the same applies

$$P\ Q = {}_wH$$

(33)

The problem is that the value ${}_wH$ does not match the

$$H = \sum_{i=1}^{n} h_i ,$$

(34)

which is the default value for the weights.

Insert one of the options for addressing the problem. [9], [10]. Let us start from the definition of weights. For all *i* is valid

$$h_i = w_i H .$$

(35)

Therefore

$$\ln(h_i) = \ln(w_i) + \ln(H)$$

(36)

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$$w_i \ln(h_i) = w_i \ln(w_i) + w_i \ln(H)$$

(37)

Adding the obtained relationship over all i, we get

$$\ln({}_wH) = \sum_{i=1}^{n} w_i \ln(w_i) + \ln(H) \sum_{i=1}^{n} w_i .$$

(38)

Since the sum of the weights is equal to one, we get

$$\ln({}_wH) = \sum_{i=1}^{n} w_i \ln(w_i) + \ln(H) \sum_{i=1}^{n} w_i .$$

(39)

Hence

$${}_wH = E\, H ,$$

(40)

where

$$E = \exp\left[\sum_{i=1}^{n} w_i \ln(w_i)\right]$$

(41)

Hence

$$H = \frac{{}_wH}{E} .$$

(42)

Consideration is illustrated on the example

Tab. 11 Results of process

n	Item	$w_i \ln(p_i q_i)$	$w_i \ln(w_i)$	$w_i \ln(H)$
1	PM_{10}	0,005211	-0,00214	0,007351909
2	SO_2	0,006741	-0,00266	0,009396060
3	NO_x	0,009047	-0,00339	0,012439815
4	CO_2	28,320567	-0,00103	28,32159609
Σt		28,341566	-0,00922	28,35078387
EXP(Σ)		2 035 099 290 843	0,990824	2 053 945 864 520

Easy to see that

$$\mathbf{2\ 053\ 945\ 864\ 520 = \frac{2\ 035\ 099\ 290\ 843}{0,990824}}$$

From the relation

$$H = \frac{{}_wH}{E} .$$

(43)

we get

$$H = \frac{P\, Q}{E} .$$

(44)

Since P is the proportion indicator, it is advisable to perform a correction for indicator Q. So get decomposition

$$H = P\, \frac{Q}{E} .$$

(45)

It can however be selected for the correction value P E_P correction value Q E_Q, so that it applies

$$E = E_P\ E_Q .$$

(46)

The numerical value is obtained for P=18 975
and Q=107 254 308.

If P is not corrected, the corrected Q is equal to 108 247 565,04 as is true

$$\mathbf{2\ 053\ 945\ 864\ 520 = 18\ 975 * \frac{107\ 254\ 308}{0,990824}}$$

(47)

$$\mathbf{2\ 053\ 945\ 864\ 520 = 18\ 975 * 108\ 247\ 565,04 .}$$

(48)

III. CONCLUSION

The article indicated one of the possible methods of obtaining overall characteristics. Commonly used features are analyzed in terms of their explanatory power and in terms of accuracy. On this issue article implicitly points.

Generally, when examining structured variables, it is necessary to examine the influence of factors weighing on aggregate variables. This must include in the survey and related analyzes.

Next viewpoints should be considered concern the sensitivity to changes, and the characteristics of the variables that determine them

Formulas used in the description and definition of the characteristics might be expressing by different way e.g. d ln(x) or statements dx / x have the same meaning.

This article draws attention to other aspects of description and quantification of phenomena and processes. The result should be considered a rough estimate with advantages and disadvantages of aggregate indicator.

These possibilities are illustrated on the example of problematic quantification of environmental damage. This procedure could help to improve the quality and acceleration of data for decision-making processes

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