

AIR POLLUTION ASSESSMENT USING INTUITIONISTIC HIERARCHICAL FUZZY INFERENCE SYSTEMS

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Abstract: *The paper presents the basic notions of intuitionistic fuzzy sets and intuitionistic fuzzy inference systems. Further, there is a design of intuitionistic hierarchical fuzzy inference system of Mamdani type for air pollution assessment. The proposed approach is based on the fact that air pollution assessment is rather subjective. Therefore, a generalization of fuzzy logic seems to be suitable for air pollution assessment.*

Keywords: *Air Pollution, Air Quality, Intuitionistic Fuzzy Logic, Intuitionistic Fuzzy Inference System.*

1. Introduction

Current approaches to air pollution assessment consider relevant air pollutants (e.g. CO, NO₂, O₃, PM₁₀ and SO₂) frequently monitored at long-term stations within air pollution monitoring networks. However, due to inconsistency and distinction of each air pollutant, there is a vagueness or fuzziness in air quality. Fuzziness makes the use of sharp boundaries in classification schemes hard to justify. Moreover, different breakpoint concentration values and air quality standards are reported in the literature [12,28]. Further, it would be significant to consider local conditions when defining breakpoint concentrations. As a matter of fact, different areas of the world are characterised by different climatic conditions influencing the effect of atmospheric pollutants on human health and also the response of population to air pollution could be different. This fuzziness led some environmental researchers to look for advanced assessment methods based on fuzzy logic [13,21,22,23], such as fuzzy synthetic evaluation [24]. Another issue is coupled to the fact that there are several pollutants presented simultaneously in the atmosphere. Therefore, the effects on human health due to the simultaneous presence of different pollutants in the atmosphere should be considered.

Based on presented facts it is obvious that a generalization of fuzzy logic [15] would be suitable for air pollution assessment. At this time there are several generalizations of fuzzy set theory for various objectives. Intuitionistic fuzzy sets (IFSs) theory [5,6,7,8,9,27] represents one of the generalizations, the notion introduced by K. T. Atanassov [2,3]. Intuitionistic fuzzy sets theory has been applied in different areas, for example logic programming [4], multiattribute decision-making processes [18], reasoning [19], optimization in an intuitionistic fuzzy environment [1], medical diagnosis [10], etc. Another possibility of its application is the modelling of economic, social and environmental processes. The IFSs are for example also suitable for the air quality modelling as they provide a good description of object attributes by means of membership functions and non-membership functions. They also present a strong possibility to express uncertainty.

The paper presents the basic notions of IFSs, intuitionistic fuzzy inference systems (IFISs) and a design of intuitionistic hierarchical fuzzy inference systems (IHFISs) of Mamdani type. Hereby, it points out the reduction of IF – THEN rules. Based on [20], the output of IFIS is defined in general. In the next part of the paper, we design and formalize of the tree IHFIS of

Mamdani type. The classification of the i -th district $o_i^t \in O$, $O = \{ o_1^t, o_2^t, \dots, o_i^t, \dots, o_n^t \}$ in time t to the j -th class $\omega_{i,j}^t \in \Omega$, $\Omega = \{ \omega_{1,j}^t, \omega_{2,j}^t, \dots, \omega_{i,j}^t, \dots, \omega_{n,j}^t \}$ realized by IHFIS assists state administration to evaluate air quality. The knowledge of notable experts in the field of air quality measuring gives support to the results of the classification.

2. Problem Formulation

A well-known air pollution assessment approach is the air quality index (AQI) developed by Environmental Protection Agency (EPA) [11]. It is defined with respect to the five main common pollutants, i.e. CO, NO₂, O₃, PM₁₀ and SO₂. An aggregate AQI based on the combined effects of five criteria pollutants (CO, SO₂, NO₂, O₃ and PM₁₀) taking into account the European standards was developed by [17]. The AQI uses both the direct numerical expression and the linguistic description. The values of air pollutants are transformed into a dimensionless number characterizing the state of air pollution. Based on the value of the AQI the state of air pollution can be classified into six classes. A sample of classes $\omega_{i,j}^t \in \Omega$ for the AQI of the Czech National Institute of Public Health (CNIPH) is presented in Table 1.

Table 1 AQI classes $\omega_{i,j}^t \in \Omega$ of the CNIPH

AQI	$\omega_{i,j}^t$	Class description
$\langle 0,1 \rangle$	$\omega_{i,1}^t$	Clean air, very healthy environment.
$\langle 1,2 \rangle$	$\omega_{i,2}^t$	Satisfactory air, healthy environment.
$\langle 2,3 \rangle$	$\omega_{i,3}^t$	Slightly polluted air, acceptable environment.
$\langle 3,4 \rangle$	$\omega_{i,4}^t$	Polluted air, environment dangerous for sensitive population.
$\langle 4,5 \rangle$	$\omega_{i,5}^t$	High polluted air, environment dangerous for the whole population.
$\langle 5,6 \rangle$	$\omega_{i,6}^t$	Very high polluted air, harmful environment.

Another AQI used in the Czech Republic was developed by the Czech Hydro meteorological Institute. The AQI is based on the results of weight concentrations measures of substances in the air (Table 2). The evaluation takes the possible influence of human health into account [26].

Table 2 AQI of the Czech Hydrometeorological Institute

Air quality	SO ₂	NO ₂	CO	O ₃	PM ₁₀
	1h [$\mu\text{g}\cdot\text{m}^{-3}$]		8h [$\mu\text{g}\cdot\text{m}^{-3}$]	1h [$\mu\text{g}\cdot\text{m}^{-3}$]	
Very good	0-25	0-25	0-1.10 ³	0-33	0-15
Good	25-50	25-50	1.10 ³ -2.10 ³	33-65	15-30
Favourable	50-120	50-100	2.10 ³ -4.10 ³	65-120	30-50
Satisfactory	120-250	100-200	4.10 ³ -1.10 ⁴	120-180	50-70
Bad	250-500	200-400	1.10 ⁴ -3.10 ⁴	180-240	70-150
Very bad	>500	>400	>3.10 ⁴	>240	>150

Except the air pollutants, there are also other components influencing the overall air pollution. For example, solar radiation, the speed or the direction of wind, air humidity and air pressure represent these components. The interaction of both types of parameters (air pollutants and meteorological parameters) can cause an increase of air pollution and influence the human health this way. The design of the parameters, based on previous correlation analysis and recommendations of notable experts, can be realized as presented in Table 3.

Parameters	
Air pollutants	$x_1^t = \text{SO}_2$, SO_2 is sulphur dioxide.
	$x_2^t = \text{O}_3$, O_3 is ozone.
	$x_3^t = \text{NO}$, NO_2 (NO_x) are nitrogen oxides.
	$x_4^t = \text{CO}$, CO is carbon monoxide.
	$x_5^t = \text{PM}_{10}$, PM_{10} is particulate matter (dust).
Meteorological	$x_6^t = \text{SW}$, SW is the speed of wind.
	$x_7^t = \text{DW}$, DW is the direction of wind.
	$x_8^t = T_3$, T_3 is the temperature 3 meters above the Earth's surface.
	$x_9^t = \text{RH}$, RH is relative air humidity.
	$x_{10}^t = \text{AP}$, AP is air pressure.
	$x_{11}^t = \text{SR}$ is solar radiation.

Table 3 Parameters design for air pollution assessment

Based on the presented facts, the problem of air pollution assessment represents a classification of objects (districts) $o_i^t \in O$, $O = \{ o_1^t, o_2^t, \dots, o_i^t, \dots, o_n^t \}$ in time t into the j -th class $\omega_{i,j}^t \in \Omega$. The monthly values of parameters $\mathbf{x}^t = (x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t)$, $m=11$ for $o_i^t \in O$, $O = \{ o_1^t, o_2^t, \dots, o_i^t, \dots, o_n^t \}$, districts in the city of Pardubice, Czech Republic, (Fig. 1) represent the data set.



Fig. 1 The map of the districts (black points)

Legend: Bus stops: (Cihelna (CI), Dubina (DU), Polabiny (PO), Rosice (RO), Rybitví (RY), Srnojedy (SR)), crossroads: (Palacha-Pichlova (PP), Náměstí Republiky (NR)), Lázně Bohdaneč (LB), chemical factory of Paramo (PA).

3. Basic Notions of Intuitionistic Fuzzy Sets

The concept of intuitionistic fuzzy sets is the generalization of the concept of fuzzy sets, the notion introduced by L. A. Zadeh [29]. The theory of intuitionistic fuzzy sets is well suited to deal with vagueness. Recently, the intuitionistic fuzzy sets have been used to intuitionistic classification models which can accommodate imprecise information.

Let a set X be a non-empty fixed set. An intuitionistic fuzzy set A in X is an object having the form [2,3]

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}, \quad (1)$$

where the function $\mu_A: X \rightarrow [0,1]$ defines the degree of membership function and the function $\nu_A: X \rightarrow [0,1]$ defines the degree of non-membership function, respectively, of the element $x \in X$ to the set A , which is a subset of X , and $A \subset X$, respectively; moreover for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\forall x \in X$ must hold. The amount

$$\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)) \quad (2)$$

is called the hesitation part, which may cater to either membership value or non-membership value, or both. For each intuitionistic fuzzy set in X , we will call $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ as the intuitionistic index of the element x in set A . It is a hesitancy degree of x to A . It is obvious that $0 \leq \pi_A(x) \leq 1$ for each $x \in X$. The value denotes a measure of non-determinancy. The intuitionistic indices $\pi_A(x)$ are such that the larger $\pi_A(x)$ the higher a hesitation margin of the decision maker. Intuitionistic indices allow us to calculate the best final results (and the worst one) we can expect in a process leading to a final optimal decision. Next we define an accuracy function H to evaluate the degree of accuracy of IF-set by the form $H(A) = \mu_A(x) + \nu_A(x)$, where $H(A) \in [0,1]$. From the definition H , it can be also expressed as follows $H(A) = \mu_A(x) + \nu_A(x) = 1 - \pi_A(x)$. The larger value of $H(A)$, the more the degree of accuracy of the IF-set A .

If A and B are two intuitionistic fuzzy sets of the set X , then [2,3]

$$A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(v_A(x), v_B(x)) \rangle \mid x \in X \}, \quad (3)$$

$$A \cup B = \{ \langle x, \max(\mu_A(x), \mu_B(x)), \min(v_A(x), v_B(x)) \rangle \mid x \in X \}, \quad (4)$$

$$A \subset B \text{ iff } \forall x \in X, (\mu_A(x) \leq \mu_B(x) \text{ and } v_A(x) \geq v_B(x)), \quad (5)$$

$$A \supset B \text{ iff if } B \subset A, \quad (6)$$

$$A = B \text{ iff } \forall x \in X, (\mu_A(x) = \mu_B(x) \text{ and } v_A(x) = v_B(x)), \quad (7)$$

$$\bar{A} = \{ \langle x, v_A(x), \mu_A(x) \rangle \mid x \in X \}. \quad (8)$$

4. Intuitionistic Hierarchical Fuzzy Inference Systems Design

Let there exists a general IFIS defined in [20]. Then it is possible to define its output y_η as

$$y_\eta = (1 - \pi_A(x)) \times y_\mu + \pi_A(x) \times y_v, \quad (9)$$

where y_μ is the output of the FIS using the membership function $\mu_A(x)$, and y_v is the output of the FIS using the non-membership function $v_A(x)$.

Let x_1, x_2, \dots, x_m be input variables, and let $y_\eta^{1,1}, y_\eta^{2,1}, \dots, y_\eta^{q,1}$ be the outputs of subsystems $FIS_\eta^{1,1}, FIS_\eta^{1,2}, \dots, FIS_\eta^{q,1}$, where $\eta = \mu$ are membership functions ($\eta = v$ are non-membership functions). Then, IF – THEN rules $R^{h_{1,1}}, R^{h_{2,1}}, \dots, R^{h_{q,1}}$ of the tree IHFIS, presented in Fig. 2, where q is the number of layers, can be defined as follows:

$$\text{Layer 1: } FIS_\eta^{1,1} \ R^{h_{1,1}} : \text{IF } x_1 \text{ is } A_1^{h_{1,1}} \text{ AND } x_2 \text{ is } A_2^{h_{1,1}} \text{ THEN } y_\eta^{1,1} \text{ is } B^{h_{1,1}}, \dots \quad (10)$$

$$\text{Layer 2: } FIS_\eta^{1,2} \ R^{h_{2,1}} : \text{IF } y_\eta^{1,1} \text{ is } B^{h_{1,1}} \text{ AND } y_\eta^{1,2} \text{ is } B^{h_{1,2}} \text{ THEN } y_\eta^{2,1} \text{ is } B^{h_{2,1}}, \dots \quad (11)$$

$$\text{Layer } q: \quad FIS_\eta^{q,1} \ R^{h_{q,1}} : \text{IF } y_\eta^{q-1,1} \text{ is } B^{h_{q-1,1}} \text{ AND } y_\eta^{q-1,2} \text{ is } B^{h_{q-1,2}} \text{ THEN } y_\eta^{q,1} \text{ is } B^{h_{q,1}}, \quad (12)$$

where $h_{1,1} = h_{1,2} = \dots = h_{q,u} = \{1, 2, \dots, k^m\}$, $u=1, 2$, $A_1^{h_{1,1}}, A_2^{h_{1,1}}, \dots, A_n^{h_{q,1}}$ are linguistic variables corresponding to fuzzy sets represented as $\eta_1^{h_{1,1}}(x_i), \eta_2^{h_{1,1}}(x_i), \dots, \eta_m^{h_{q,1}}(x_i)$, $B^{h_{1,1}}, B^{h_{1,2}}, \dots, B^{h_{q,1}}$ are linguistic variables corresponding to fuzzy sets represented as $\eta^{h_{1,1}}(y_\eta^{1,1}), \eta^{h_{1,2}}(y_\eta^{1,2}), \dots, \eta^{h_{q,1}}(y_\eta^{q,1})$, and $\eta_{B^{h_{1,1}}}(y_j^{1,1}), \eta_{B^{h_{1,2}}}(y_j^{1,2}), \dots, \eta_{B^{h_{q,1}}}(y_j^{q,1})$ are membership function $\eta = \mu$ (non-membership function $\eta = v$) values of aggregate fuzzy set for outputs $y_j^{1,1}, y_j^{1,2}, \dots, y_j^{q,1}$.

The outputs $y_\eta^{1,1}, y_\eta^{1,2}, \dots, y_\eta^{q,1}$ of particular subsystems $FIS_\eta^{1,1}, FIS_\eta^{1,2}, \dots, FIS_\eta^{q,1}$ of the tree IHFIS can be expressed by using defuzzification method Center of Gravity (COG) [16]. The outputs of particular subsystems $FIS_\eta^{1,1}, FIS_\eta^{1,2}, \dots, FIS_\eta^{q,1}$ in each layer of the IHFIS are calculated as follows

$$y_\eta^{r,s}(B^{h_{r,s}}) = (1 - \pi_\mu^{r,s}) \times y_\mu^{r,s}(B^{h_{r,s}}) + \pi_\nu^{r,s} \times y_\nu^{r,s}(B^{h_{r,s}}), \text{ for } r = 1, 2, \dots, q, s = 1, 2. \quad (13)$$

Similarly, it is possible to design and define a cascade IHFIS, and various other, hybrid IHFISs.

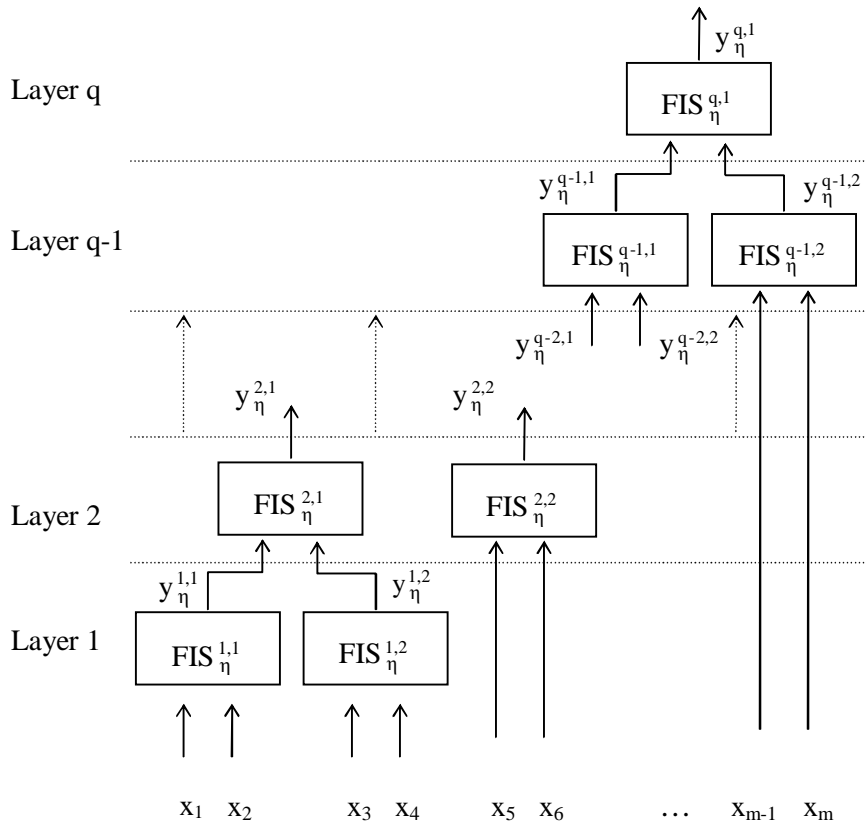


Fig. 2 A tree IHFIS

5. Air Pollution Assessment using Intuitionistic Hierarchical Fuzzy Inference System and Analysis of the Results

Air pollution assessment represents a classification problem. As such it can be realized by the tree IHFIS with inputs (parameters) $\mathbf{x}^t = (x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t)$, $m=11$, outputs $y_{\eta}^{1,1}, y_{\eta}^{1,2}, \dots, y_{\eta}^{q,1}$ of individual subsystems $FIS_{\eta}^{1,1}, FIS_{\eta}^{1,2}, \dots, FIS_{\eta}^{q,1}$, $q=6$. The design of specific tree IHFIS results from the recommendation of experts in given field. Thus, it simulates their decision-making process. The design of input (output) membership functions μ and non-membership functions ν is realized by means of clustering algorithms. As an example, the input membership functions μ for input parameters x_1^t and x_2^t of the particular subsystem $FIS_{\mu}^{1,1}$ are presented in Fig. 3. In a similar manner, the input non-membership functions ν for input parameters x_1^t and x_2^t of the subsystem $FIS_{\nu}^{1,1}$ are presented in Fig.4. These functions are designed for an example of intuitionistic index $\pi=0.05$.

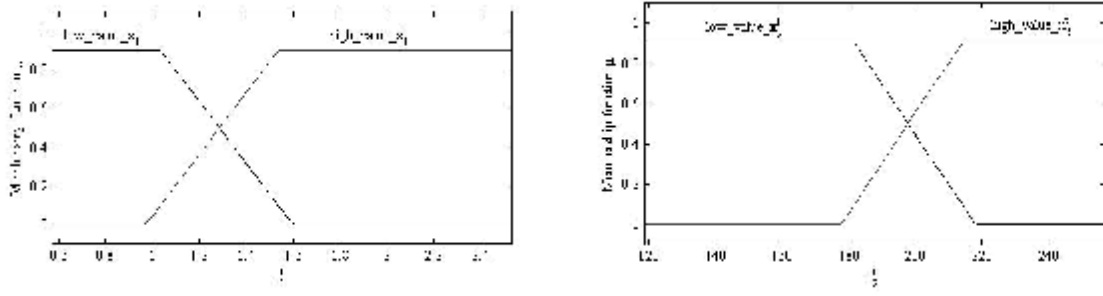


Fig. 3 Input membership functions μ for input parameters x_1^t and x_2^t of subsystem $FIS_u^{1,1}$

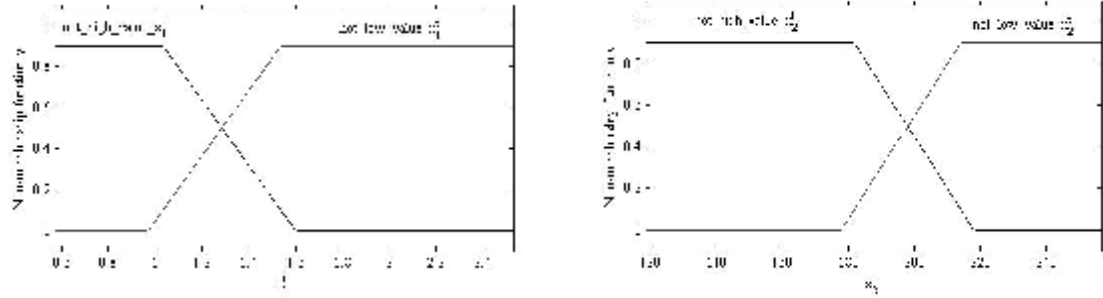


Fig. 4 Input non-membership functions ν for input parameters x_1^t and x_2^t of subsystem $FIS_v^{1,1}$

Base of IF – THEN rules for subsystem $FIS_{\eta}^{1,1}$ is defined as:

for $FIS_{\mu}^{1,1}$ (14)

$FIS_{\mu}^{1,1}$ R¹: if x_1^t is low_value_ x_1^t AND x_2^t is low_value_ x_2^t

then $y_{\mu}^{1,1}$ is low_value_ $y_{\mu}^{1,1}$,

$FIS_{\mu}^{1,1}$ R²: if x_1^t is low_value_ x_1^t AND x_2^t is high_value_ x_2^t

then $y_{\mu}^{1,1}$ is medium_value_ $y_{\mu}^{1,1}$,

$FIS_{\mu}^{1,1}$ R³: if x_1^t is high_value_ x_1^t AND x_2^t is low_value_ x_2^t

then $y_{\mu}^{1,1}$ is medium_value_ $y_{\mu}^{1,1}$,

$FIS_{\mu}^{1,1}$ R⁴: if x_1^t is high_value_ x_1^t AND x_2^t is high_value_ x_2^t

then $y_{\mu}^{1,1}$ is high_value_ $y_{\mu}^{1,1}$.

for $FIS_v^{1,1}$ (15)

$FIS_v^{1,1}$ R¹: if x_1^t is not_low_value_ x_1^t AND x_2^t is not_low_value_ x_2^t

then $y_v^{1,1}$ is not_low_value_ $y_v^{1,1}$,

$FIS_v^{1,1}$ R²: if x_1^t is not_low_value_ x_1^t AND x_2^t is not_high_value_ x_2^t

then $y_v^{1,1}$ is not_medium_value_ $y_v^{1,1}$,

$FIS_v^{1,1}$ R³: if x_1^t is not_high_value_ x_1^t AND x_2^t is not_low_value_ x_2^t

then $y_v^{1,1}$ is not_medium_value_ $y_v^{1,1}$,

$FIS_v^{1,1}$ R⁴: if x_1^t is not_high_value_ x_1^t AND x_2^t is not_high_value_ x_2^t

then $y_v^{1,1}$ is not_high_value_ $y_v^{1,1}$.

The inference mechanism of particular subsystems $FIS_{\eta}^{1,1}, FIS_{\eta}^{1,2}, \dots, FIS_{\eta}^{q,1}$ involves also the process of implication (MIN method) and aggregation (MAX Method) within IF – THEN rules, and the process of defuzzification by COG method of obtained outputs to the crisp values. The output $y_{\eta}^{6,1}$ of the designed tree IHFIS (the frequencies f of the classes $\omega_{i,j}^t \in \Omega$ is presented in Fig. 5a. The classification of the i -th district $o_i^t \in O, O = \{o_1^t, o_2^t, \dots, o_i^t, \dots, o_n^t\}$ in time t to the j -th class $\omega_{i,j}^t \in \Omega, \Omega = \{\omega_{1,j}^t, \omega_{2,j}^t, \dots, \omega_{i,j}^t, \dots, \omega_{n,j}^t\}, (n=60), j=1,2,3$ by the tree IHFIS using association index is shown in Fig. 5b. Yearly average values (2001-2006) of measurements are used for the assessment of ten districts in Pardubice micro region.

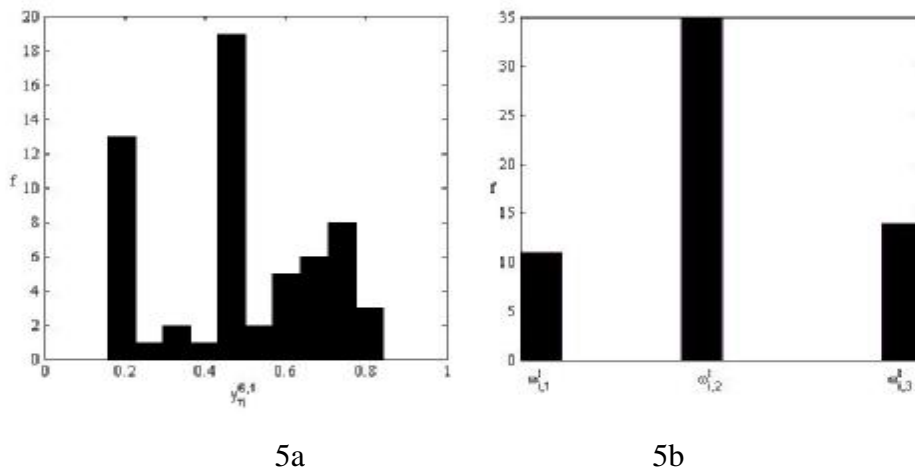


Fig. 5a The output $y_{\eta}^{6,1}$ of the tree IHFIS (the frequencies f of the classes)

Fig. 5b Classification of the district $o_i^t \in O$ into classes $\omega_{i,j}^t \in \Omega, j=1,2,3$ (the frequencies f of the classes) using association index ξ

6. Conclusion

Air pollution assessment is currently realized by methods combining mathematical-statistical methods and expert opinion. They have some limitations which can be overcome using IFSs. Intuitionistic fuzzy sets make it possible to process uncertainty and the expert knowledge. The concept of the IFS can be viewed as an alternative approach to define a fuzzy set. In general, the theory of IFSs is the generalization of fuzzy sets. Therefore, it is expected that IFSs could be used to simulate human decision-making and classification processes and any activities requiring human expertise and knowledge, which are inevitably imprecise or not totally reliable. Based on IFSs, the paper presents the design of tree IHFIS of Mamdani type. The IHFIS defined this way works more effectively than the HFIS [14,25] as it provides stronger possibility to accommodate imprecise information and, at the same time, the number of IF – THEN rules is reduced compared to the IFIS. Output of the IHFIS uses the theory of general IFIS presented in [20]. The introduction of association index ξ makes it possible to point out the classification of the i -th district $o_i \in O$ into the j -th class $\omega_{i,j} \in \Omega$ realized by the tree IHFIS initially. The gained results represent the recommendations for the state administration of the city of Pardubice in the field of air quality development. They can also serve as a basis for the municipal crisis management in crisis situations. The model was carried out in programme environment MATLAB/Simulink under MS Windows XP operation system.

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