CLASSIFICATION ON THE BASIS INTUITIONISTIC HIERARCHICAL FUZZY INFERENCE SYSTEM OF MAMDANI TYPE

Vladimír Olej, Petr Hájek

Univerzita Pardubice, Fakulta ekonomicko-správní, Ústav systémového inženýrství a informatiky

Abstract: *The paper presents basic notions of intuitionistic fuzzy sets and fuzzy inference systems of Mamdani type for the consecutive design of intuitionistic hierarchical fuzzy inference systems. On the basis of the given concept, we design and formalize tree intuitionistic hierarchical fuzzy inference systems of Mamdani type. Moreover, we present a classification problem realization for an example of municipal creditworthiness modelling.*

Keywords: *Intuitionistic fuzzy sets, fuzzy inference systems, intuitionistic hierarchical fuzzy inference systems, municipal creditworthiness modelling*

1. Introduction

The paper presents the basic notions of intuitionistic fuzzy sets [1], [2] and compares the number of IF-THEN rules in fuzzy inference system (FIS) [3] with the number of IF-THEN rules in hierarchical fuzzy inference system (HFIS) of Mamdani type [4]. Hereby, it points out the reduction of IF-THEN rules. Further, it presents one type of the HFIS, namely a tree HFIS. Based on [5], the output of intuitionistic fuzzy inference system (IFIS) is defined in general. In the next part of the paper, we design and formalize intuitionistic hierarchical fuzzy inference systems (IHFIS) of Mamdani type. Moreover, the classification of the i-th municipality $o_i \in O$, $O=\{o_1,o_2, \dots ,o_i, \dots ,o_n\}$ to the j-th class $\omega_{i,j} \in \Omega$, $\Omega=\{\omega_{1,j},\omega_{2,j}, \dots ,\omega_{i,j}, \dots ,\omega_{n,j}\}$ by the IHFIS presented in the paper assists state administration to the municipal creditworthiness evaluation.

2. Basic Notions

The concept of intuitionistic fuzzy sets is the generalization of the concept of fuzzy sets, the notion introduced by L. A. Zadeh [6]. The theory of intuitionistic fuzzy sets is well suited to deal with vagueness. Recently, the intuitionistic fuzzy sets have been used to intuitionistic classification models which can accommodate imprecise information.

Let a set X be a non-empty fixed set. An intuitionistic fuzzy set A in X is an object having the form [1], [2]

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}, \tag{1}$$

where the function $\mu_A:X \to [0,1]$ defines the degree of membership function and the function $\nu_A:X \to [0,1]$ defines the degree of non-membership function, respectively, of the element $x \in X$ to the set A, which is a subset of X, and $A \subset X$, respectively; moreover for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$, $\forall x \in X$ must hold. The amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the hesitation part, which may cater to either membership value or non-membership value, or both. For each intuitionistic fuzzy set in X, we will call $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ as the intuitionistic index of the element x in set A. It is a hesitancy degree of x to A. It is obvious that $0 \le \pi_A(x) \le 1$ for each $x \in X$. If A and B are two intuitionistic fuzzy sets of the set X, then [1], [2]

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle \mid x \in X\}, \tag{2}$$

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle \mid x \in X\}, \tag{3}$$

$A \subset B$ iff $\forall x \in X$, $(\mu_A(x) \leq \mu_B(x))$ and $(\nu_A(x) \geq \nu_B(x))$, (4)

$A \supset B$ iff if $B \subset A$, (5)

$A = B$ iff $\forall x \in X$, $(\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x))$, (6)

$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$. (7)

Let there exists the FIS of Mamdani type defined in [3]. Then the number p_{FIS} of IF-THEN rules is as follows

$$p_{FIS} = k^n, \tag{8}$$

where: – k is the number of membership functions,
– n is the number of input variables.

For a great number n of input variables, the FIS of Mamdani type may be inefficient due to the increase in the number p_{FIS} of IF-THEN rules. One of the ways to reduce the number p_{FIS} of IF-THEN rules is to design the FIS of Mamdani type with a hierarchical structure [4]. Reducing the number p_{FIS} of IF-THEN rules leads to a reduction in computing demand of the system. This way, it comes to be more effective.

Let there exists the HFIS of Mamdani type defined in [4]. Then the number p_{HFIS} of IF-THEN rules is given as

$$p_{HFIS} = \left(\frac{n-t}{t-1} + 1\right) \times k^t, \tag{9}$$

where t is the number of variables in each layer.

The minimum number p_{HFIS} of IF-THEN rules is achieved if each subsystem in the HFIS has only 2 inputs (t=2). Then it represents a trivially solvable problem. If t=2, then $p_{HFIS} = (n - 1) \times k^2$. There exist two basic types of the HFIS [4], that is a tree and a cascade HFIS. Based on these types of the HFIS, various other (hybrid) HFIS can be designed which are suitable for modelling.

Let there exists a general IFIS defined in [5]. Then it is possible to define its output y_η as

$$y_\eta = (1 - \pi_A(x)) \times y_\mu + \pi_A(x) \times y_\nu, \tag{10}$$

where: – y_μ is the output of the FIS using the membership function $\mu_A(x)$,
– y_ν is the output of the FIS using the non-membership function $\nu_A(x)$.

3. Tree Intuitionistic Hierarchical Fuzzy Inference Systems of Mamdani Type Design

Let $x_1, x_2, \ldots, x_n$ be input variables, and let $y_\eta^{1,1}, y_\eta^{2,1}, \ldots, y_\eta^{q,1}$ be the outputs of subsystems $FIS_\eta^{1,1}, FIS_\eta^{1,2}, \ldots, FIS_\eta^{q,1}$, where $\eta=\mu$ are membership functions ($\eta=\nu$ are non-membership functions).

Then, IF-THEN rules $R^{h_{1,1}}, R^{h_{2,1}}, \ldots, R^{h_{q,1}}$ of the tree IHFIS, presented in Fig. 1, where q is the number of layers, can be defined as follows:

Layer 1: $FIS_\eta^{1,1}$ $R^{h_{1,1}}$: IF x_1 is $A_1^{h_{1,1}}$ AND x_2 is $A_2^{h_{1,1}}$ THEN $y_\eta^{1,1}$ is $B^{h_{1,1}}$,

$FIS_\eta^{1,2}$ $R^{h_{1,2}}$: IF x_3 is $A_3^{h_{1,2}}$ AND x_4 is $A_4^{h_{1,2}}$ THEN $y_\eta^{1,2}$ is $B^{h_{1,2}}$,

Layer 2: $FIS_\eta^{2,1}$ $R^{h_{2,1}}$: IF $y_\eta^{1,1}$ is $B^{h_{1,1}}$ AND $y_\eta^{1,2}$ is $B^{h_{1,2}}$ THEN $y_\eta^{2,1}$ is $B^{h_{2,1}}$,

$FIS_\eta^{2,2}$ $R^{h_{2,2}}$: IF x_5 is $A_5^{h_{2,2}}$ AND x_6 is $A_6^{h_{2,2}}$ THEN $y_\eta^{2,2}$ is $B^{h_{2,2}}$,

. . . (11)

Layer q: $FIS_\eta^{q,1}$ $\quad R^{h_{q,1}}$: IF $y_\eta^{q-1,1}$ is $B^{h_{q-1,1}}$ AND $y_\eta^{q-1,2}$ is $B^{h_{q-1,2}}$ THEN $y_\eta^{q,1}$ is $B^{h_{q,1}}$,

where: – $h_{1,1} = h_{1,2} = \ldots = h_{q,1} = \{1,2, \ldots ,k^2\}$,

– $A_1^{h_{1,1}}, A_2^{h_{1,1}}, \ldots, A_n^{h_{q,1}}$ are linguistic variables corresponding to fuzzy sets represented as $\eta_1^{h_{1,1}}(x_i), \eta_2^{h_{1,1}}(x_i), \ldots, \eta_n^{h_{q,1}}(x_i)$,

– $B^{h_{1,1}}, B^{h_{1,2}}, \ldots, B^{h_{q,1}}$ are linguistic variables corresponding to fuzzy sets represented as $\eta^{h_{1,1}}(y_\eta^{1,1}), \eta^{h_{1,2}}(y_\eta^{1,2}), \ldots, \eta^{h_{q,1}}(y_\eta^{q,1})$,

– $\eta_{B^{h_{1,1}}}(y_j^{1,1}), \eta_{B^{h_{1,2}}}(y_j^{1,2}), \ldots, \eta_{B^{h_{q,1}}}(y_j^{q,1})$ are membership function η=μ (non-membership function η=ν) values of aggregate fuzzy set for outputs $y_j^{1,1}, y_j^{1,2}, \ldots, y_j^{q,1}$.

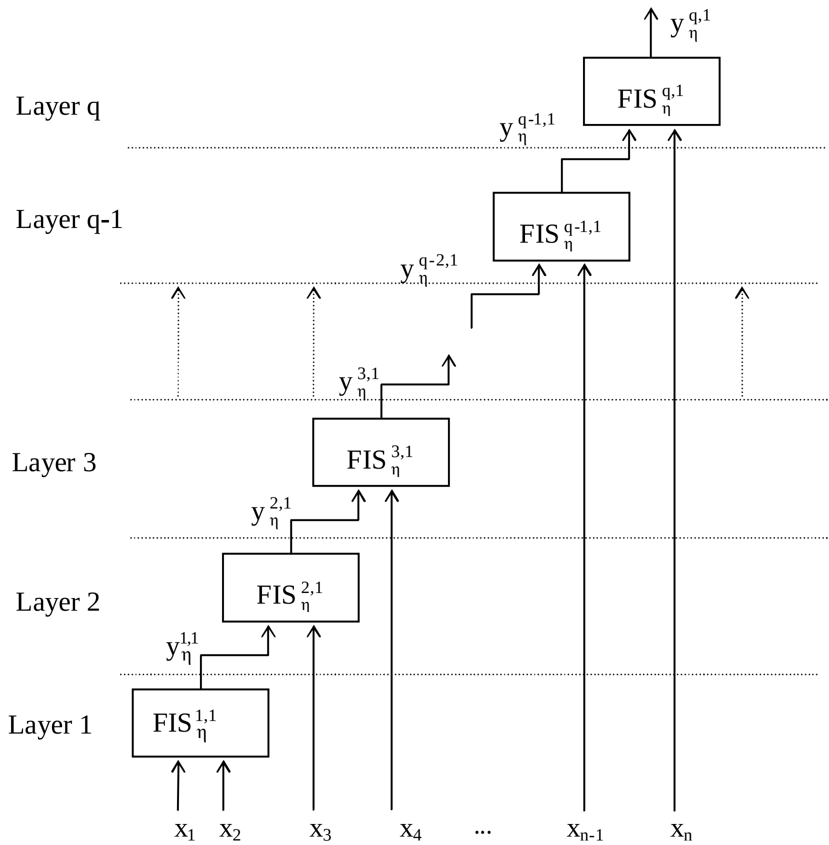

Fig. 1 A tree IHFIS

The outputs $y_\eta^{1,1}, y_\eta^{2,1}, \ldots, y_\eta^{q,1}$ of particular subsystems $FIS_\eta^{1,1}, FIS_\eta^{1,2}, \ldots, FIS_\eta^{q,1}$ of the tree IHFIS can be expressed by using defuzzification method Center of Gravity [3] and the outputs of particular subsystems $FIS_\eta^{1,1}, FIS_\eta^{1,2}, \ldots, FIS_\eta^{q,1}$ in each layer of the IHFIS are calculated as follows

$$y_\eta^{r,s}(B^{h_{r,s}}) = (1-\pi_\mu^{r,s}) \times y_\mu^{r,s}(B^{h_{r,s}}) + \pi_n^{r,s} \times y_n^{r,s}(B^{h_{r,s}}), \text{ for } r = 1,2, \ldots ,q,\ s = 1,2. \qquad (12)$$

4. Municipal Creditworthiness Classification

In [7], [8], [9], [10] there are mentioned common categories of parameters, namely economic, debt, financial, and administrative categories. Economic parameters affect long-term credit risk. The municipalities with more diversified economy and more favourable socio-economic conditions are better prepared for the economic recession. Debt parameters include the size and structure of the debt. Financial parameters inform about the budget implementation. Their values are extracted from the municipality budget. The design of parameters and classes of municipal creditworthiness, can be realized as presented in Tab. 1, and Tab. 2. Based on the presented facts, the following data matrix **P** can be designed

$$\mathbf{P} = \begin{array}{c|ccccc:c} & x_1 & \dots & x_k & \dots & x_m & \omega \\ \hline o_1 & x_{1,1} & \dots & x_{1,k} & \dots & x_{1,m} & \omega_{1,j} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ o_i & x_{i,1} & \dots & x_{i,k} & \dots & x_{i,m} & \omega_{i,j} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ o_n & x_{n,1} & \dots & x_{n,k} & \dots & x_{n,m} & \omega_{n,j} \end{array},$$

where $o_i \in O$, $O=\{o_1,o_2, \dots ,o_i, \dots ,o_n\}$ are objects (municipalities), x_k is the k-th parameter, $x_{i,k}$ is the value of the parameter x_k for the i-th object $o_i \in O$, $\omega_{i,j} \in \Omega$ is the j-th class assigned to the i-th object $o_i \in O$, $\mathbf{p}_i =(x_{i,1},x_{i,2}, \dots ,x_{i,k}, \dots ,x_{i,m})$ is the i-th pattern, $\mathbf{x}=(x_1,x_2, \dots ,x_k, \dots ,x_m)$ is the parameters vector.

Tab. 1: Municipal creditworthiness parameters design

Parameters	
Economic	$x_1 = PO_r$, PO_r is the population in the r-th year.
	$x_2 = PO_r/PO_{r-s}$, PO_{r-s} is the population in the year r-s, and s is the selected time period.
	$x_3 = U$, U is the unemployment rate in a municipality.
	$x_4 = \sum_{i=1}^{k}(PZO_i/PZ)^2$, PZO_i is the employed population of the municipality in the i-th economic sector, i=1,2, … ,k, PZ is the total number of employed inhabitants, k is the number of the economic sector.
Debt	$x_5 = DS/OP$, $x_5 \in <0,1>$, DS is debt service, OP are periodical revenues.
	$x_6 = CD/PO$, CD is a total debt.
	$x_7 = KD/CD$, $x_7 \in <0,1>$, KD is short-term debt.
Financial	$x_8 = OP/BV$, $x_8 \in R^+$, BV are current expenditures.
	$x_9 = VP/CP$, $x_9 \in <0,1>$, VP are own revenues, CP are total revenues.
	$x_{10} = KV/CV$, $x_{10} \in <0,1>$, KV are capital expenditures, CV are total expenditures.
	$x_{11} = IP/CP$, $x_{11} \in <0,1>$, IP are capital revenues.
	$x_{12} = LM/PO$, [Czech Crowns], LM is the size of the municipal liquid assets.

Municipal creditworthiness modelling [7], [8], [9], [10] represents a classification problem. By the defining the problem in this manner it is possible for it to be modelled by unsupervised methods (if classes $\omega_{i,j} \in \Omega$ are not known). Municipal creditworthiness modelling is realized by the tree IHFIS. The input (output) membership functions μ (non-membership functions ν) for input parameters x_1 and x_2 ($y_{\mu}^{1,1}$) of the particular subsystem $FIS_{\mu}^{1,1}$ ($FIS_{n}^{1,1}$) for modelling was designed. These functions are designed for an example of intuitionistic index π=0.05.

Tab. 2: Municipal creditworthiness classes

Class $\omega_{i,j}$, j=1,2,3	Description
$\omega_{i,1}$	High ability of a municipality to meet its financial obligation. Very favorable economic conditions, low debt, and excellent budget implementation.
$\omega_{i,2}$	Good ability of a municipality to meet its financial obligation. A municipality with stable economy, medium debt, and good budget implementation.
$\omega_{i,3}$	Municipality meets its financial obligation with difficulty, and only under favorable economic conditions.

The output $y_{\eta}^{6,1}$ of the designed tree IHFIS (the frequencies f of the classes $\omega_{i,j}\in\Omega$) is presented in Fig. 2. The classification problem works with the set of input patterns $\mathbf{p}_i$ assigned to one of the classes $\omega_{i,j}\in\Omega$. The classifier chooses one of the classes $\omega_{i,j}\in\Omega$ for the given pattern $\mathbf{p}_i$. The classification of the set of municipalities $O=\{o_1,o_2, \dots ,o_i, \dots ,o_n\}$, n=452, into classes $\omega_{i,j}\in\Omega$, $\Omega=\{\omega_{1,j},\omega_{2,j}, \dots ,\omega_{i,j}, \dots ,\omega_{n,j}\}$, j=1,2,3, by the IHFIS using association index is shown in Fig. 3. By means of intuitionistic index π it is possible to calculate association index ξ. Association index $\xi=\mu-\nu\times\pi$ [11] emphasizes high values of the membership function μ (association) and reduces low values of the non-membership function ν (non-association). Based on the analysis of the association index ξ it is possible to classify the i-th municipality $o_i\in O$ into the j-th class $\omega_{i,j}\in\Omega$, $\Omega=\{\omega_{1,j},\omega_{2,j}, \dots ,\omega_{i,j}, \dots ,\omega_{n,j}\}$, j=1,2,3, in the microregion of Pardubice, the Czech Republic.

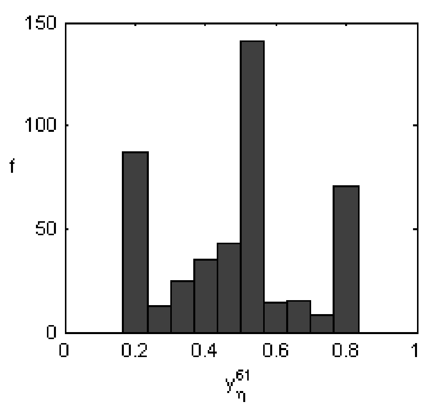

Fig. 2 The ouptut $y_{\eta}^{6,1}$ of the tree IHFIS

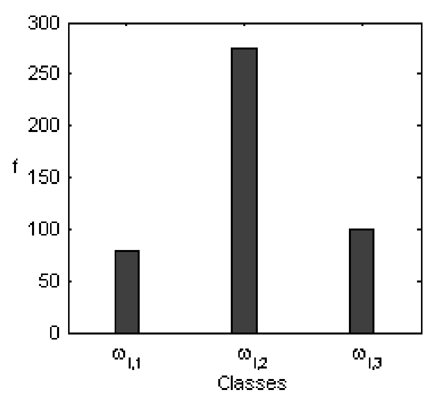

Fig. 3 Classification of the municipalities using association index ξ

5. Conclusion

Based on intuitionistic fuzzy sets, the paper presents the design of tree and cascade IHFIS of Mamdani type. The IHFIS defined this way works more effective than the HFIS [4] as it provides stronger possibility to accommodate imprecise information and, at the same time, the number of IF-THEN rules is reduced compared to the IFIS. Output of the IHFIS uses the theory of general IFIS presented in [5]. The introduction of association index ξ makes it possible to point out the classification of the i-th municipality $o_i\in O$ into the j-th class $\omega_{i,j}\in\Omega$ realized by the tree IHFIS initially. The gained results represent the recommendations for the state administration of the city of Pardubice in the field of the municipal creditworthiness development. They can also serve as a basis for the municipal crisis management in crises situations. The model was carried out in programme environment MATLAB/Simulink under MS Windows XP operation system.

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Kontaktní adresa:

prof. Ing. Vladimír Olej, CSc., Ing. Petr Hájek, Ph.D.
Ústav systémového inženýrství a informatiky
Fakulta ekonomicko-správní
Univerzita Pardubice
Studentská 84, 532 10 Pardubice
email: vladimir.olej@upce.cz, petr.hajek@upce.cz
tel.: +420 466 036 004

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