SCIENTIFIC PAPERS
OF THE UNIVERSITY OF PARDUBICE
Series A
Faculty of Chemical Technology
13 (2007)

MODELLING AND TITO PREDICTIVE CONTROL OF LABORATORY SYSTEM

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Received June 5, 2007

Laboratory pilot-plant is designed and realized as a system for process control education and research – e.g. mathematical modelling, experimental identification, controller design, real-time control. Key features of laboratory hydraulic-pneumatic system are described in the paper. Nonlinear mathematical-physical model of the process and its linearized state-space realization are briefly presented. TITO predictive controller is designed (from linearized model) as a control application example. Simulated and real control experiments are compared to verify the model quality. Nonlinear model contains emulation of typical process noises and disturbances and can be treated as an "image of real process" or benchmark test process.

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Introduction

Simulation and laboratory experiments have significant importance in control engineering education. Deterministic simulation is an excellent tool for control theory practicing and algorithms testing. On the other hand, experiments with real systems open additional problems and tasks (e.g., static and dynamic properties of sensors and actuators, uncertainties in process model, unmeasurable disturbances, noises, input and state variables constraints, hardware and software tools for connection between controlled plant and computer etc.). Laboratory plants bring users closer to the practical problems. From this point of view the nonlinear models including such behaviour as typical process nonlinearities, disturbances and noises represent intermediate stage between an ideal simulation and the reality. Advantage is that it is possible to simulate, try out and compare very fast different control methods and study, e.g., controller robustness.

Hydraulic-Pneumatic Process

The laboratory hydraulic-pneumatic system (HPS) was designed and realized at the University of Pardubice, Department of Process Control and Computer Techniques. It includes a combination of hydraulic and pneumatic components. The pneumatic loops create cross coupling between both classical hydraulic sections and form a multivariable system with non-typical behaviour.

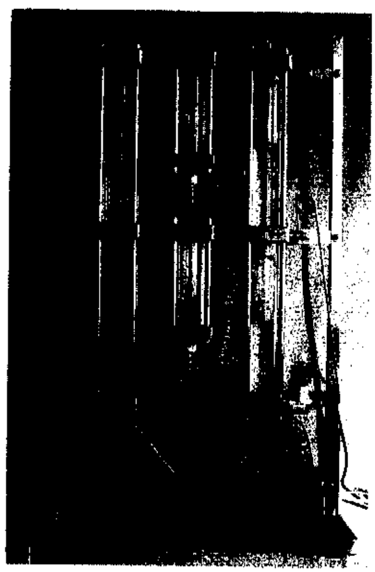

Fig. 1 Hydraulic-pneumatic laboratory system

The laboratory plant is described in detail in [1-3]. Its main parts are four cylindrical water tanks (Figs 1 and 2). The tanks are grouped in two sections. Water is pumped by two pumps into upper tanks, flows into lower tanks and from here through orifices back into the reservoir. Air spaces above the water levels are connected together by pneumatic volumes and by manually-set valves. Orifices in air chambers serve as a connection between pneumatic volumes and atmosphere. These pneumatic loops create cross coupling between both sections. The system structure and its behaviour may be manually changed by means of the size of orifices and by valves setting in the pneumatic loops.

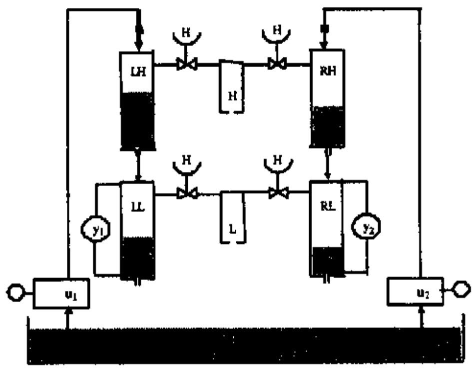

Fig. 2 Scheme of hydraulic-pneumatic process

The levels in lower tanks are measured indirectly by the difference pressure sensors. Output of pressure sensors (y_1, y_2) is given in a form of electric voltage in the range of 0-10 V. Water flows are controlled by pumps input power. Input signal for every pump (u_1, u_2) is voltage in the range of 0-10 V, which is changed into the range of 4-10 V and amplified in pump unit.

First Principle Process Model

The nonlinear model of HPS was derived on the basis of physical laws and system construction [1]. Models of the water tanks can be described by

a) mass balance based on law of mass conservation

$$Q_1 = Q_2 + \rho S \frac{dh}{dt} \tag{1}$$

where Q_1 is water inlet flow rate [kg s^{-1}], Q_2 is water outlet flow rate [kg s^{-1}], ρ is water mass density [kg m^{-3}], S is cross-section area of tank [m^2] and h is water level [m].

b) Bernoulli equation (water outlet flow rate is given as)

$$Q_2 = ks\sqrt{2\rho}\sqrt{h\rho g + p_1 - p_2} \tag{2}$$

where k is discharge coefficient, s is cross-section area of orifice [m^2], g is acceleration of gravity [m s^{-2}], p_1 is pressure above the water level [Pa] and p_2 is pressure under the orifice [Pa].
In Table I variables used in Eqs (1) and (2) are specified (according to Fig. 2).

Table I Denotation of variables for water tanks

Tank	Q_1	Q_2	S	k	s	h	p_1	p_2
LH	Q_L	Q_{LH}	S_L	k_L	s_L	h_{LH}	p_A	p_L
RH	Q_R	Q_{RH}	S_R	k_R	s_R	h_{RH}	p_A	p_L
LL	Q_{LH}	Q_{LL}	S_L	k_L	s_L	h_{LL}	p_L	p_A
RL	Q_{RH}	Q_{RL}	S_R	k_R	s_R	h_{RL}	p_L	p_A

Only the lower pneumatic loop is considered in this model (pneumatic volume C_H is open into atmosphere). The lower pneumatic loop was modelled on the basis of

a) mass balance equivalent of Eq. (1)

$$0 = Q_{CL} + \frac{d(V_L\rho_L)}{dt} \tag{3}$$

where Q_{CL} is air outlet flow rate [kg s^{-1}], V_L is pneumatic volume (air chamber volume plus volume above water levels) [m^3] and ρ_L is air mass density [kg m^{-3}].

b) equivalent of Eq. (2), which was simplified into the following form

$$Q_{CL} = k_{CL}s_{CL}(p_L - p_A) \tag{4}$$

where k_{CL} is air discharge coefficient [s m^{-1}], s_{CL} is cross-section area of orifice [m^2], p_L is pressure in the pneumatic loop [Pa] and p_A is atmospheric pressure [Pa].

c) equation of gas state

$$p_L = \rho_L rT \tag{5}$$

where r is specific gas constant [J K^{-1} kg^{-1}] and T is air temperature [K].

Equations (3), (4) and (5) may be combined together into relationship

$$\frac{d(\rho_L V_L)}{dt} = -k_{CL} s_{CL} r T (p_L - p_A) \tag{6}$$

The pump static characteristic is considered in the form $Q = a(u - u_0)^b$, where Q is water flow-rate [kg s^{-1}], u is input signal for the pump unit [V] (u_0 [V] is the signal corresponding to zero flow rate), a and b are the pump specific coefficients.

The pressure sensor static characteristic is in the form $y = ch + d$, where y is output signal from the pressure sensor [V], h is water level [m] c and d are the pressure sensor specific coefficients.

Water discharge coefficients k_L, k_R, air discharge coefficient k_{CL} and coefficients for pump and pressure sensor static characteristic (a, b, u_0, c, d) were estimated experimentally. The model has five state variables - four water levels and pressure in the lower pneumatic loop, two inputs - input signals for pump unit u_L and u_R and two outputs - output signals from pressure sensors y_L and y_R. Furthermore, the nonlinear model includes emulation of typical process noises and disturbances.

Linearized Model

The linearization of nonlinear model for given steady state point was realized by the Taylor expansion where the second and higher order terms were omitted [1]. Symbol Δ denotes variable deviation from steady state, *e.g.* $\Delta h = h - h_0$, where subscript 0 denotes steady state. The steady state of pressure in lower pneumatic loop is atmospheric pressure p_A.

State space model has the following form

$$\begin{bmatrix} \frac{d\Delta h_{LH}}{dt} \\ \frac{d\Delta h_{RH}}{dt} \\ \frac{d\Delta p_L}{dt} \\ \frac{d\Delta h_{LL}}{dt} \\ \frac{d\Delta h_{RL}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_L} & 0 & \frac{Z}{T_L} & 0 & 0 \\ 0 & -\frac{1}{T_R} & \frac{Z}{T_R} & 0 & 0 \\ \frac{Z_{hL}}{T_p} & \frac{Z_{hR}}{T_p} & -\frac{1}{T_p} & -\frac{Z_{hL}}{T_p} & -\frac{Z_{hR}}{T_p} \\ \frac{1}{T_L} & 0 & -\frac{2Z}{T_L} & -\frac{1}{T_L} & 0 \\ 0 & \frac{1}{T_R} & -\frac{2Z}{T_R} & 0 & -\frac{1}{T_R} \end{bmatrix} \cdot \begin{bmatrix} \Delta h_{LH} \\ \Delta h_{RH} \\ \Delta p_L \\ \Delta h_{LL} \\ \Delta h_{RL} \end{bmatrix} + \begin{bmatrix} \frac{Z_{QL}}{T_L} & 0 \\ 0 & \frac{Z_{QR}}{T_R} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta u_L \\ \Delta u_R \end{bmatrix} \tag{7}$$

$$\begin{bmatrix}\Delta y_{\mathrm{L}}\\ \Delta y_{\mathrm{R}}\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & c_{\mathrm{L}} & 0\\ 0 & 0 & 0 & 0 & c_{\mathrm{R}}\end{bmatrix} \cdot \begin{bmatrix}\Delta h_{\mathrm{LH}}\\ \Delta h_{\mathrm{RH}}\\ \Delta p_{\mathrm{L}}\\ \Delta h_{\mathrm{LL}}\\ \Delta h_{\mathrm{RL}}\end{bmatrix} \tag{8}$$

where time constants are

$$T_{\mathrm{L}} = \frac{S_{\mathrm{L}} Q_{\mathrm{L}}}{\rho g k_{\mathrm{L}}^2 s_{\mathrm{L}}^2},\ T_{\mathrm{R}} = \frac{S_{\mathrm{R}} Q_{\mathrm{R}}}{\rho g k_{\mathrm{R}}^2 s_{\mathrm{R}}^2},\ T_p = \frac{V_{\mathrm{L}} Q_{\mathrm{L}} Q_{\mathrm{R}}}{2 p_{\mathrm{A}} \left(k_{\mathrm{L}}^2 s_{\mathrm{L}}^2 Q_{\mathrm{R}} + k_{\mathrm{R}}^2 s_{\mathrm{R}}^2 Q_{\mathrm{L}}\right) + Q_{\mathrm{L}} Q_{\mathrm{R}} r T k_{\mathrm{CL}} s_{\mathrm{CL}}}$$

and gains are

$$Z = \frac{1}{\rho g},\ Z_{\mathrm{QL}} = \frac{a_{\mathrm{L}} b_{\mathrm{L}} \left(u_{\mathrm{L}} - u_{0\mathrm{L}}\right)^{b_{\mathrm{L}}-1} T_{\mathrm{L}}}{\rho S_{\mathrm{L}}},\ Z_{\mathrm{QR}} = \frac{a_{\mathrm{R}} b_{\mathrm{R}} \left(u_{\mathrm{R}} - u_{0\mathrm{R}}\right)^{b_{\mathrm{R}}-1} T_{\mathrm{R}}}{\rho S_{\mathrm{R}}}$$

$$Z_{h\mathrm{L}} = \frac{p_{\mathrm{A}} S_{\mathrm{L}} T_p}{V_{\mathrm{L}} T_{\mathrm{L}}},\ Z_{h\mathrm{R}} = \frac{p_{\mathrm{A}} S_{\mathrm{R}} T_p}{V_{\mathrm{L}} T_{\mathrm{R}}}$$

TITO Predictive Control

Controlled variables are output signals from pressure sensors (lower tanks water levels), manipulated variables are input signals for pump unit (water flow rates). The predictive controller computes control actions with the use of linear process model (9), actual state $\mathbf{x}(k)$ and future set-points knowledge $\mathbf{w}$. Control actions are optimal to the quadratic criterion (11). Future control error (difference between predicted and desired plant output) and control increments are penalized in the criterion. If we consider no constraints and time invariant process model, an analytical solution is possible (12). We get relationship for the optimum future control actions for whole control horizon but only actual control action is realized and calculation is repeated in next sampling time (receding horizon strategy is applied). The controller has the form of Eq. (13), where $\mathbf{K}$ and $\mathbf{F}$ are matrices ($\mathbf{K}$ is submatrix from $\mathbf{L}$). State observer (Kalman estimator) is used because only two variables from state vector are measured (only lower water levels).
State-space model:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned} \tag{9}$$

Future plant output prediction:

$$\mathbf{y} = \mathbf{H}\mathbf{u} + \mathbf{F}\mathbf{x}(k),\ \mathbf{F} = f(\mathbf{A},\mathbf{C}),\ \mathbf{H} = g(\mathbf{A},\mathbf{B},\mathbf{C}) \tag{10}$$

Criterion:

$$J = \left(\mathbf{F}\mathbf{x}(k) + \mathbf{H}\mathbf{u} - \mathbf{w}\right)^T \mathbf{Q}\left(\mathbf{F}\mathbf{x}(k) + \mathbf{H}\mathbf{u} - \mathbf{w}\right) + \mathbf{u}^T\mathbf{R}\mathbf{u} \tag{11}$$

Future optimal control actions:

$$\mathbf{u} = \left(\mathbf{H}^T\mathbf{Q}\mathbf{H} + \mathbf{R}\right)^{-1}\mathbf{H}^T\mathbf{Q}\left(\mathbf{w} - \mathbf{F}\mathbf{x}(k)\right) = \mathbf{L}\left(\mathbf{w} - \mathbf{F}\mathbf{x}(k)\right) \tag{12}$$

Actual control action - linear control law:

$$\mathbf{u}(k) = \mathbf{K}\left(\mathbf{w} - \mathbf{F}\mathbf{x}(k)\right) \tag{13}$$

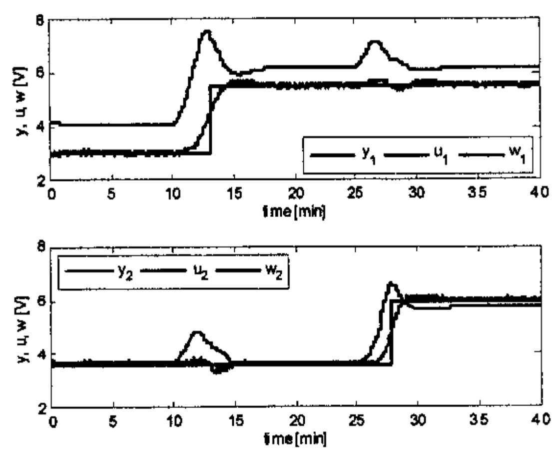

Fig. 3 Simulated control experiment

Conclusion

The nonlinear model of the laboratory system has been derived. Unknown parameters were estimated from experimental data. State-space linear model of the system is used by TITO predictive controller. First closed-loop control experiments are simulated with nonlinear model (see Fig. 3). Consequently real system is controlled (Fig. 4). Both control experiments are similar, which prove model quality. Many different parameters and constants exceed the possibilities of this paper and are not specified — the aim was to introduce a laboratory system and mathematical model and outline predictive control design. The authors offer to send mentioned models as a TITO control simulation benchmark system in form of Simulink schemes and MATLAB scripts by E-mail.

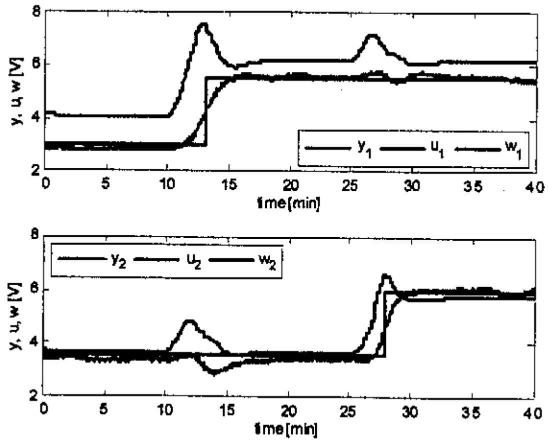

Fig. 4 Real control experiment

Acknowledgements

This project was supported by Grant Agency of the Czech Republic under grant No. 102/03/0625 and by the research project MSM 0021627505 of Ministry of Education, Youth and Sports.

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