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WALL EFFECTS ON THE TERMINAL VELOCITY OF A SINGLE NON-SPHERICAL PARTICLE MOVING THROUGH A NON-NEWTONIAN FLUID

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The wall effects on the terminal falling velocity of short rigid cylinders and square prisms moving through non-Newtonian polymer solutions of different degree of pseudoplasticity and elasticity have been examined experimentally in the creeping flow region. In the experiments, terminal falling velocities of particles were measured in Perspex cylindrical test columns of various diameters. The wall effect results have been evaluated and the relationships based on the power law and Carreau viscosity models have been proposed for prediction of the wall effect correction factor for non-spherical particles and cylindrical confining walls.

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Introduction

It is known that the confining walls or bounding surfaces cause an extra retardation force on a particle falling through a viscous fluid. The interaction of the particle with walls depends on the particle shape, orientation, and position, as well as on the fluid rheological behaviour, flow regime, and the geometry of the containing walls. Owing to this interaction, the particle terminal velocity is reduced in comparison with that reached in the infinite medium. The particle retardation is customarily quantified using the wall correction factor F_W, which can be defined as the ratio of the terminal falling velocity of a particle in a bounded fluid to that in an unbounded one

$$F_W = \frac{u_t}{u_{t\infty}} \tag{1}$$

Since the theoretical determination of the wall factor is a complicated problem [1], the great deal of information on wall effects available in literature, especially for non-Newtonian fluids, concerns spherical particles and is based on experiments [2,3]. Up to now, however, very little effort has been devoted to the investigation of wall effects on terminal velocity of non-spherical particles falling through non-Newtonian fluids [4,5]. In this paper, the results are reported of our experimental investigations of the wall effects on terminal velocity of short cylinders and prisms falling slowly through non-Newtonian polymer solutions.

Experimental

The terminal velocities were measured of cylinders and square prisms falling through glycerol and water solutions of polymers in cylindrical columns. The experiments were performed in the creeping flow region.

Particles

Twenty four types of particles made of duralumin ($0.2 \leq h/d \leq 5$, $\rho_s = 2800$ kg m^{-3}) [6], 26 types of square prisms made of aluminium ($0.25 \leq c/a \leq 5$, $\rho_s = 2638$ kg m^{-3}), and 26 types of particles of the same geometry made of PVC ($\rho_s = 1379$ kg m^{-3}) were used in our terminal velocity measurements.

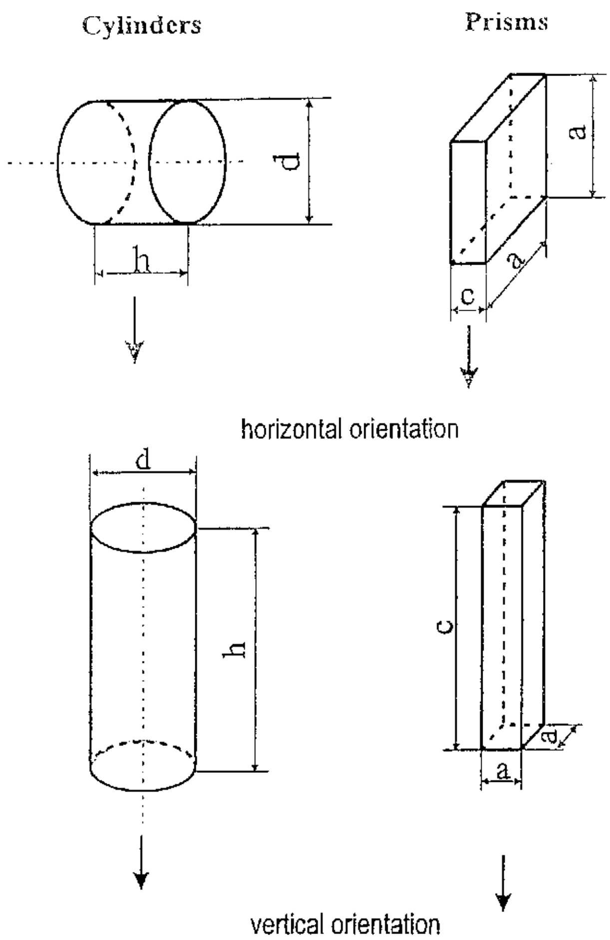

Fig. 1 Horizontal and vertical orientations of the fall of cylinders and prisms tested

Liquids

The test liquids included glycerol and water solutions of methylcellulose Tylose MH 4000, hydroxyethylcellulose Natrosol 250 MR, poly(ethylene oxide) Polyox WSR 301, and polyacryl-amide Kerafloc A 4008. The shear rate-shear stress dependencies were measured using the rotary cylindrical rheometer Rheotest II. Additional viscosity measurements and dynamic tests with oscillating stresses were carried out on the Haake rheometer RS 150.

Test Vessels

Five Perspex cylindrical columns of diameters D = 190, 140 mm, 80 mm, 40 mm, and 20 mm were used in our particle settling experiments. The height of the columns was approximately 1 m.

Terminal Falling Velocity Measurements

The measurements of terminal velocities of particles were performed for two basic orientations of particles designated as vertical and horizontal (see Fig. 1). For the vertical orientation of cylindrical particles, the cylinder axis of revolution is identical with that of the test column; for the horizontal orientation, the particle axis of revolution is perpendicular to the test column longitudinal axis. For the vertical orientation of prisms, the square base of particle is perpendicular to the test column longitudinal axis (to the direction of fall of particle); the horizontal orientation is characterised by perpendicularity of a side wall of prism to the particle fall direction.

During a particle fall through Newtonian liquid (glycerol), the both orientations vertical and horizontal were stable. During fall through non-Newtonian liquids, only one orientation was stable, nearly vertical one for particles characterised by the ratio h/d (or c/a) > 1 and nearly horizontal one for h/d (or c/a) < 1.

The terminal falling velocities were determined from the mean time of particle fall through two test column sections of 10 cm length. The stop watch reading to 0.01 s was used for timing the particles. From our measurements, 1250 experimental data of terminal falling velocity u_t have been obtained in the range of Reynolds number $Re_{nt} \cong Re_{Ct}$ from 0.001 to 0.3.

Results and Discussion

Rheological Characteristics of Test Liquids

From the rheometric data measured, parameters n and K of the power-law

$$\eta(\dot{\gamma}) = K\dot{\gamma}^{n-1} \tag{2}$$

and parameters η_0, λ and m of the Carreau model

Table I Density and power-law parameters of the test liquids, 20 °C

Liquid	ρ, kg m^{-3}	n	K, Pa s^n	$\dot{\gamma}$, s^{-1}	Particles
Glycerol	1260	1.000	1.4	-	
Tylose 1.7%	1003	0.863	1.24	1.5 – 27	
Natrosol 1.6%	1003	0.697	2.28	1.5 – 27	cylinders
Polyox 0.8%	1001	0.527	0.956	1.5 – 27	
Kerafloc 0.3%	1000	0.309	1.98	1.5 – 27	
Glycerol	1260	1.000	1.422	-	
Natrosol 1.6%	1003	0.848 0.713	1.840 2.095	0.37 – 1.46 1.5 – 27	rectangular prisms
Kerafloc 0.3%	1000	0.311	2.477	1.5 – 81	

Table II Carreau model parameters of the test liquids, 20 °C

Liquid	Carreau model parameters			Particles
	η_0, Pa s	λ, s	m	
Glycerol	1.42	0.000	1.000	
Tylose 1.7%	1.20	0.238	0.764	
Natrosol 1.6%	2.3	0.349	0.534	cylinders
Polyox 0.8%	2.1	4.09	0.491	
Kerafloc 0.3%	10.0	10.6	0.313	
Glycerol	1.42	0.000	1.000	
Natrosol 1.6%	2.1	0.591	0.638	rectangular prisms
Kerafloc 0.3%	12.0	9.16	0.306	

$$\eta(\dot{\gamma}) = \frac{\eta_0}{\left[1 + (\lambda\dot{\gamma})^2\right]^{(1-m)/2}} \tag{3}$$

were determined. Since the simple power-law model describes the viscometric data only over a limited interval, the parameters K and n have been calculated in the shear rate range downward from the level of shear rate specified by the value

$\dot{\gamma} = 3u_t/d_v$. This value is considered to be the maximum relevant value of shear rate. The examples of parameters of viscosity models are given in Tables I and II for test liquids.

Solutions of glycerol exhibit Newtonian behaviour, polymer solutions exhibit a different degree of shear thinning and elasticity. The solution of Tylose is slightly shear thinning, the solution of Natrosol exhibits a greater measure of shear thinning and solutions of Polyox, and Kerafloc are highly shear thinning. It followed from additional dynamic tests that along with the growth of shear thinning the degree of polymer solution elasticity increases as well. At the same time, it was confirmed that the degree of elasticity of polymer solutions can be roughly evaluated and compared according to value λ of time parameter of the Carreau flow model (Table II).

Wall Factor

The extent of wall effects on retardation of particle terminal velocity was evaluated according to values of the wall factor F_W defined by Eq. (1). The terminal falling velocity u_t of each particle was determined as a ratio of the length of the measuring section and the particle falling time. The corresponding values of velocities $u_{t\infty}$ were obtained by means of extrapolation of experimental dependencies of u_t on the ratio d_v/D_e. The effective diameter D_e of the column with a falling particle was defined as

$$D_e = D - z \qquad (4)$$

where z is the longest dimension of the particle projection into a plane perpendicular to the direction of particle fall. The effective diameter D_e fulfils the limiting requirements: $F_W = 1$ for $z \ll D_e$ and $F_W = 0$ for $z = D$.

An example of experimental dependence of F_W upon d_v/D_e is shown in Fig. 2. In order to make this figure more clear, the experimental data measured in the solution of Tylose, which lie among the data obtained for glycerol and solution of Natrosol, were not plotted. In spite of scattering the data displayed, a dependence of the value of the wall factor F_W on rheological behaviour of the fluid is evident. Like in the fall of spherical particles through non-Newtonian polymer solutions (e.g. [7]), wall effects are decreasing with increasing pseudoplasticity and elasticity of the fluid. At the same time, the values of F_W for the fall of particles through nearly inelastic solutions of Tylose and Natrosol are evidently not different from those for the Newtonian glycerol.

Analysing the experimental data, we conclude that the relationship,

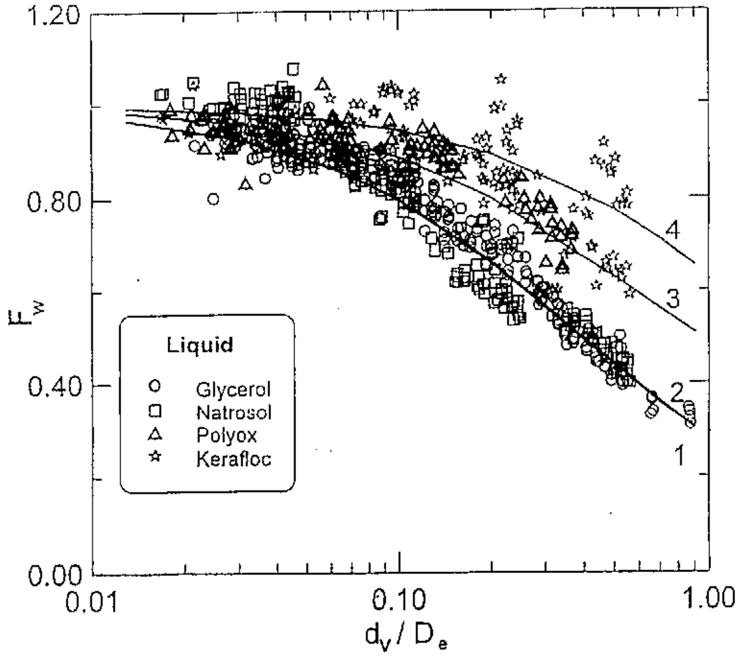

Fig. 2 Dependence of the wall correction factor F_W upon the ratio d_v/D_e. Full lines correspond to Eqs (5) – (7): 1 – Eq. (6), ○ Glycerol; 2 – Eq. (6), □ Natrosol; 3 – Eq. (7), △ Polyox; 4 – Eq. (7), ☆ Kerafloc

$$F_W = \frac{1}{1 + c\dfrac{d_v}{D_e}} \tag{5}$$

with the coefficient c depending on the rheological properties of fluid, can be used for correlation of the wall factor F_W.

Using the power-law viscosity model (2), the following relationships were found for the expression of dependence of the coefficient c on the fluid power-law index n:

$$\text{for} \quad 0.7 \le n \le 1 \qquad c(n) = 2.37 - 0.206n \tag{6}$$

and

for $n < 0.7 \qquad c(n) = 3.7n^{1.58}$ (7)

If the Carreau viscosity model (3) is used for representation of rheometric data, the dependence of the coefficient c on the Carreau model parameter m and on the dimensionless time parameter

$$\Lambda = \frac{2\lambda u_{t\infty}}{d_v} \tag{8}$$

must be considered. In this case, the following relationship

$$c(m, \Lambda) = 2.60\left[1 - (1 - m)^2 \Lambda^{0.12}\right] \tag{9}$$

has been obtained by a non-linear regression of experimental data of F_W [8].

The agreement between our experimental wall factor data and those calculated according to Eq. (5) using Eqs (6) or (7) is evident from Fig. 2, in which the predictions corresponding to Eqs (6) or (7) are given by full lines. The value of the mean relative deviation

$$\delta_m = \frac{1}{p}\sum_{i=1}^{p}\left|\frac{F_{W,c}}{F_W} - 1\right|\cdot 100 \tag{10}$$

between experimental F_W data and the $F_{W,c}$ data calculated according to Eq. (5) using Eqs (6) or (7) was 5.1% for all experiments ($p = 1250$). At the same time, the value of δ_m for glycerol was 4.8% and for polymer solutions 5.4%.The mean relative deviation δ_m between all experimental values of F_W and those calculated according to Eq. (5) using Eq. (9) was also 5.1%, for glycerol was 4.9% and for polymer solutions was 5.3%. It is evident that the accuracy of both the relationships (6), (7) and (9) is nearly the same. In this respect, the use of more complicated Eq. (9) based on the Carreau viscosity model is not necessary for the wall correction factor prediction if the power-law parameters K and n are determined in an appropriate interval of shear rates.

Conclusion

The terminal falling velocities of short cylinders and squared prisms have been measured in Newtonian solution of glycerol and non-Newtonian polymer solutions contained in cylindrical vessels of various diameter.

It was verified that the retardation effect of the walls on the terminal falling velocity of particles decreases with the increasing pseudoplasticity and elasticity of the fluids. At the same time, the values of the wall correction factor F_W for the fall of particles through nearly inelastic and moderately shear thinning polymer solutions are not evidently different from those for Newtonian glycerol. Simple relationships based on power-law and Carreau viscosity models have been proposed for the prediction of the wall factor F_W in the investigated applications and creeping flow regime. Considering nearly the same accuracy of relationships proposed, the use of more complicated equation based on the Carreau viscosity model is not necessary for the wall correction factor predictive calculation.

Acknowledgements

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Symbols

- a side of prism square cross-section, m
- c length of prism, m
- c coefficient in Eq. (5)
- d_v equal volume sphere diameter, m
- D diameter of test column, m
- D_e effective diameter of the test column with falling particle defined by Eq. (4), m
- F_W wall effect correction factor defined by Eq. (1)
- h height of cylinder, m
- K power-law parameter (consistency coefficient), Pa s^n
- m Carreau model parameter
- n power-law parameter (flow index)
- Re_{Ct} Reynolds number for the fall of a particle through a Carreau model fluid $\left(= \frac{d_v u_{t\infty} \rho}{\eta_0} \left(1 + \frac{1}{4}\Lambda^2 \right)^{\frac{1-m}{2}} \right)$

Re_{nt} power-law Reynolds number for the fall of a particle $\left(= \frac{d_v^n u_{t\infty}^{2-n} \rho}{K} \right)$

u_t particle terminal falling velocity, m s^{-1}

$u_{t\infty}$ particle terminal falling velocity in unbounded liquid, m s^{-1}

z longest dimension of particle projection into a plane perpendicular to the direction of motion, m

$\dot{\gamma}$ shear rate, s^{-1}

δ_m mean relative deviation

η viscosity of non-Newtonian liquid, Pa s

η_0 parameter of the Carreau viscosity model (zero shear rate viscosity), Pa s

λ parameter of the Carreau viscosity model, s

Λ dimensionless time, Eq. (8)

ρ fluid density, kg m^{-3}

ρ_s particle density, kg m^{-3}

Subscripts

c calculated

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