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APPROXIMATIVE SOLUTION OF THE MOMENTUM TRANSFER IN INCOMPRESSIBLE NEWTONIAN FLUID-FLUIDIZED BED SYSTEM

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A method based on the dividing of the particle resistance into the frictional and shape components is suggested for the calculation of incompressible Newtonian fluid-fluidized bed system of spherical particles. This method led to the relationship which includes the general aspects of relationships of both the Richarson–Zaki and Wen–Yu types. The form of the dependence of a ratio of shape to frictional resistance on the Reynolds number has been determined experimentally. A comparison is made with a set of relationships given in the literature.

Introduction

In our previous papers [1–3] we dealt with the solution of flow of Newtonian and non-Newtonian fluids through beds of particles and of fall of a single particle in non-

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Newtonian fluids. The difference from earlier ways of solving these problems consisted in that the resistance of both the single spherical particle and spherical particles in the bed appearing in the momentum balance was divided into the frictional and shape components. This led to introduction of a new dimensionless quantity (denoted as ψ) given by the ratio of the shape and frictional resistances which is called the resistance number.

For creeping flow of Newtonian fluids (NF) and generalized Newtonian fluids (GNF) through fixed and fluidized beds of spherical particles the value of resistance number is the same as in the Stokes′ approach for a single spherical particle [1,2] (ψ = 1/2). In the region of manifestation of Reynolds number the forms of dependences of quantity ψ on the Reynolds number for both the fixed and fluidized beds must be determined experimentally.

The aim of the present study is to determine the course of the above-mentioned dependence for fluidized bed. This knowledge is a condition necessary for extending the validity range of the relationships suggested in Ref. [2] for the calculations of fluidized bed in flow of an NF and GNF on the region of manifestation of inertial forces.

Theory

The paper [2] solves the problem of momentum transfer in the system GNF-fluidized bed of spherical particles by adopting the modified Rabinowitsch–Mooney equation,

$$\dot{D}_{w,f} = \frac{3+\Omega}{\tau_{w,f}^{2+\Omega}} \int_0^{\tau_{w,f}} \tau^{1+\Omega} \dot{D}(\tau)\, d\tau \tag{1}$$

where $\dot{D}_{w,f}$ and $\tau_{w,f}$ are the consistency variables, $\dot{D}(\tau)$ is the dependence of the shear rate $\dot{D}$ on the shear stress τ, and Ω is the dimensionless characteristic which depends on the porosity ε of the fluidized bed [2].

The equation (1) can be written as

$$\dot{D}_{w,f} = \frac{\tau_{w,f}}{\mu_e} \tag{2}$$

where μ_e is the effective viscosity of the system considered. For an NF $\mu_e = \mu$, where μ is the dynamic viscosity.

Using the transformation of consistency variables for flow of an NF and GNF through fixed bed of spherical particles, Eqs. (3) and (4) were obtained for consistency variables for fluidized bed [2]

$$\dot{D}_{w,f} \equiv \frac{2u_{ch,b}}{l_{ch}} = \frac{2uM_{w,f}}{d}\varepsilon^{-3.29-2.18\log(1+\psi_f)} \tag{3}$$

$$\tau_{w,f} = \frac{(\rho_s - \rho)gd\varepsilon}{6(1+\psi_f)M_{w,f}} \tag{4}$$

where the correction factor for the wall effect $M_{w,f}$ is given by the formula

$$M_{w,f} = 1 + \frac{d}{D}\varepsilon^{2.18\log(1+\psi_f)-0.02} \tag{5}$$

In Eqs. (3) – (5) $u_{ch,b} = u/\varepsilon$ is the characteristic velocity for a flow of a fluid through the beds (mean velocity in the voids), u is the superficial velocity, l_{ch} is the characteristic linear dimension of the bed, d is the diameter of spherical particle, D is the diameter of the column of circular cross-section, ρ_s is density of the particles, ρ is density of the fluid, and g is the gravitational acceleration.

The Reynolds number for a flow of an NF and GNF through the beds is given by the relationship (6)

$$Re_b = \frac{\rho u_{ch,b} l_{ch}}{\mu_e} \tag{6}$$

which after substitution for quantity l_{ch} using Eq. (3) with $u_{ch,b} = u/\varepsilon$ assumes for the fluidized bed the form

$$Re_{b,f} = \frac{\rho ud}{\mu_e M_{w,f}}\varepsilon^{1.29+2.18\log(1+\psi_f)} \tag{7}$$

For $\varepsilon = 1 (\Omega = -5/2)$ and $d/D = 0$ ($M_{w,f} = 1$) Eqs (1) – (4) and (7) assume the forms valid for a single spherical particle [3].

In analogy to the fixed bed and single particle [2,3], it is presumed that the course of dependence $\psi_f = F(Re_{b,f})$ does not depend on the rheological properties of GNF in both the creeping region of flow ($\psi_f = 1/2$) and the region of manifestation of Reynolds number. Then it is possible to apply to GNF also the form of the dependence $\psi_f = F(Re_{b,f})$ determined experimentally with NF ($\mu_e = \mu$) along with relationships (1) and (2), with that for dimensionless characteristic Ω [2], and with relationships (3) – (5) and (7).

In Ref. [2] the range of porosity and of d/D ratio (in which the relationships suggested for the creeping region of flow ($\psi_f = 1/2$) agree well with experiment) was experimetally delimited (the latter parameter up to the value $d/D < 0.083$).

Results and Discussion

When looking for a suitable form of dependence $\psi_f = F(Re_{b,f})$, we presumed that it would be similar to that valid for a single particle. Here the value of resistance number ψ_p can be calculated using some of dependences of the drag coefficient C_D

$$C_D = \frac{4(\rho_s - \rho)gd}{3\rho u_p^2} \tag{8}$$

where u_p is the fall velocity, on the Reynolds number Re_p

$$Re_p = \frac{\rho u_p d}{\mu} \tag{9}$$

given in the literature, and Eq. (10) in the form

$$\psi_p = \frac{C_D Re_p}{16} - 1 \tag{10}$$

Owing to its simplicity, Eq. (11) is used

$$C_D = \frac{24}{Re_p}(1 + aRe_p^b) \tag{11}$$

where a and b are numerical coefficients. Oseen [4] was the first to adopt the relationship of type (11) with the values of coefficients $a = 3/16$ and $b = 1$. An equation of type (11) with the values of $a = 0.15$ and $b = 0.687$ for the region of Reynolds number $0.2 < Re_p < 500 - 1000$ was also used by Schiller and Naumann [5], and with the values of $a = 0.125$ and $b = 0.72$ for $0.2 < Re_p < 1000$ by Lapple [6].

After introducing C_D from Eq. (11) in Eq. (10) and after rearranging we obtain the relationship

$$\psi_{Re,p} \equiv \psi_p - 0.5 = 1.5 a Re_p^b \equiv a' Re_p^b \qquad (12)$$

where the quantity $\psi_{Re,p}$ represents the contribution to the value of resistance number ψ_p in creeping region of flow ($\psi_p = 1/2$) due to the effect of Reynolds number.

The course of dependence $\psi_{Re,f} = F(Re_{b,f})$ was determined on the basis of our experiments using glass and lead spherical particles and aqueous glycerol solutions (Mikulášek [7]). The reliable experimental results were achieved by means of a calming section situated below the bed. A montejus has been used for the transport of the fluid through the column. For the region of higher values of Reynolds number we took the experimental data by Wilhelm and Kwauk [8], who used glass spherical particles and water. The physical properties of the systems adopted, inclusive of values of d/D ratio, are presented in Table I. A part of the results obtained with these systems is depicted in Fig. 1.

From Fig. 1 it can be seen that, within the given range of experimental values $Re_{b,f}$, the dependence $\psi_{Re,f} = F(Re_{b,f})$ can be approximated by three dependences of type (12) with different values of coefficients a' and b. First we determined values of the coefficients for the dependence corresponding to the highest range of values of Reynolds number.

In order to fix the lower limit of this dependence, we used the value $\psi_{Re,f} = 1/2$ corresponding to the first break in the dependence log $\psi_{Re,f} = F(\log Re_{b,f})$. For fixing the upper limit we took into account the fact that the relationships for consistency variables for fluidized bed were obtained by a transformation of relationships for consistency variables in fixed bed. These relationships, for unit porosity, are changed into the relationships valid for a flow through a tube involving — as a characteristic linear dimension l_{ch} of system — the hydraulic radius $r_h = D/4$ [9]. If $l_{ch} = D/4$, the Reynolds number which limits the occurrence of turbulence in the tube assumes the value

$$Re_{crit} \cong \frac{2320}{4} = 580 \qquad (13)$$

We have used this value, in analogy to fixed bed of particles [9], to delimit the laminar region of flow also for fluidized bed. From Fig. 1 it can be seen that this value agrees well with that characterising the second break in the dependence log $\psi_{Re,f} = F(\log Re_{b,f})$. The experimental results for $\psi_{Re,f} > 1/2$ and $Re_{b,f} < 580$ then allow us to derive a dependence of the following form

$$\psi_{Re,f} = 0.34 Re_{b,f}^{0.714} \tag{14}$$

which is represented in Fig. 1. From Eq. (14) with the limit value of $\psi_{Re,f} = 1/2$ it follows that $Re_{b,f} \cong 1.72$.

Comparison of Eq. (14) with relationships valid for a single particle shows that the value of exponent at the Reynolds number is greater than the value 0.687 used in the relationship by Schiller and Naumann [5]. It is, however, practically identical with the value 0.72 recommended by Lapple [6], whose relationship in the form given by Eq. (12) is represented in Fig. 1, too.

For the range of values $Re_{b,f} < 1.72$, we have presumed the value of exponent $b = 1$ like in Oseen's approach [4], and accepting the condition of $\psi_{Re,f} = 1/2$ for $Re_{b,f} = 1.72$, we have obtained the dependence in the form of Eq. (15)

$$\psi_{Re,f} = 0.29 Re_{b,f} \tag{15}$$

which is also presented in Fig. 1. Using relationship by Oseen [4], we get for the coefficient $a' = 1.5a$ ($a = 3/16$) the value of 0.28 which is practically identical with the value of 0.29 determined for fluidized bed. For the range of the highest values of Reynolds number $580 < Re_{b,f} < 1300$, the dependence derived (depicted in Fig. 1, too) thus reads as follows

$$\psi_{Re,f} = 4.0 Re_{b,f}^{0.332} \tag{16}$$

The comparison of experimental values of the quantity $1 + \psi_{f,exp}$ appearing in the relationships for both consistency variables and the values calculated from Eqs (14) – (16), experimental values of Reynolds number $Re_{b,f,exp}$ and relationship $1 + \psi_f = 1.5 + \psi_{Re,f}$ was carried out with the use of mean relative deviation δ

$$\delta = \frac{1}{N}\sum_{i=1}^{N} |\delta_{i,1}| \tag{17}$$

where the relative deviation $\delta_{i,1}$

$$\delta_{i,1} = \left[\frac{1.5 + \psi_{Re,f,exp}}{1.5 + \psi_{Re,f,calc}} - 1\right] \times 100\ \% \tag{18}$$

The values of mean relative deviation δ for the individual systems are also given in Table I. For practical purposes it is appropriate to delimit also the creeping region of flow by a certain value of $\psi_{Re,f}$ quantity comparable with experimental error. With the use of the value of the minimum deviation given in Table I ($\delta = 1.9\%$), the relation $\psi_{Re,f}/1.5 = 0.019$, and relation (15), it is then possible to determine the value $Re_{b,f} \doteq 0.1$.

Table I Properties of systems adopted and experimental results

System	d m ×10^{-3}	ρ_s kg m^{-3}	d/D	μ Pa s ×10^{-3}	ρ kg m^{-3}	N	δ %
1	1.47	2506	0.073	9.54	1158	23	1.9
2	1.47	2506	0.018	7.81	1136	22	2.8
3	1.78	2515	0.022	7.81	1136	14	3.7
4	1.78	2515	0.044	7.81	1136	16	3.9
5	1.78	2515	0.089	9.54	1158	24	3.6
6	2.55	2512	0.064	8.10	1149	14	3.2
7	1.97	11093	0.049	9.54	1158	20	3.0
8	5.21	2351	0.069	1.00	1000	29	3.0

Systems 7 and 8 in Ref. [8]

For comparison of the relationships suggested with those given in literature, we will express Eq. (2) with $\mu_e = \mu$, after the respective substitutions for consistency variables using Eqs (3) and (4) in the form

$$\frac{Ar\,\varepsilon^n}{Re_o} = 12(1 + \psi_f)M_{w,f}^2 \tag{19}$$

and the Reynolds number $Re_{b,f}$ given by Eq. (7) in the form

$$Re_{b,f} = \frac{Re_o \varepsilon^{n-3}}{M_{w,f}} \quad (20)$$

In Eqs (19) and (20) the Archimedes number Ar

$$Ar = \frac{(\rho_s - \rho)\rho g d^3}{\mu^2} \quad (21)$$

the Reynolds number Re_o

$$Re_o = \frac{u d \rho}{\mu} \quad (22)$$

and

$$n = 4.29 + 2.18 \log(1 + \psi_f) \quad (23)$$

For $M_{w,f} = 1$, $\varepsilon = 1$, when $u = u_p$, and for creeping region of flow ($\psi_f = 1/2$), the relationship (19) changes into Stokes' approach in the form valid for fall of single particle (Eq. (24) with $\psi_p = 1/2$) and the Reynolds number $Re_{b,f}$ (Eq.(20)) assumes the form valid for a single particle Re_p (Eq.(9)). Due to the different course of dependences $\psi_p = F(Re_p)$ and $\psi_f = F(Re_{b,f})$ (see Fig. 1), in the transient region the relationship for fluidized bed does not change into the form valid for a single particle (Eq. (24)).

$$\frac{Ar}{Re_p} = 12(1 + \psi_p) \quad (24)$$

For the purposes of calculation of fluidized bed in creeping region of flow, the result of Stokes' approach (Eq. (24) with $\psi_p = 1/2$) is modified with application of Eq. (25), introduced by Lewis *et al.* [10].

$$Re_p = \frac{Re_o}{\varepsilon^n} \quad (25)$$

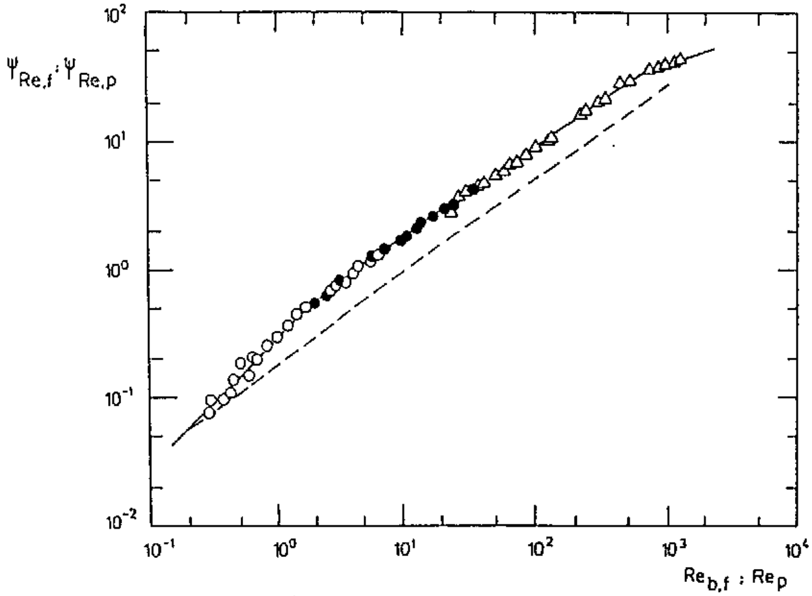

Fig. 1 The dependence of quantity ψ_{Re} on the Reynolds number Re for fluidized bed (*f*) and single paricle (*p*): ○ – glass particles-aqueous solutions of glycerol; ● – lead particles-aqueous solution of glycerol; Δ – glass particles-water (Wilhelm and Kwauk [8]); ——— Eqs (14) – (16); – – – – Eq. By Lapple [6]

Two main methods are used for calculations in the region of manifestation of the Reynolds number: that by Richardson and Zaki [11] starts from relationship (25), in which the course of dependence $n = F(Re_p)$ is determined experimentally. Similarly it is possible to consider [12] the dependences $n = F(Ar)$ or $n = F(Ar/Re_p)$.

When using the relationships valid for a single particle, it is possible to arrive at the relationships of the second method (Wen and Yu [13]) in the following way. After substitution of Re_p in Eq. (24) with the use of Eq. (25), some of the dependences $Ar/Re_p = F(Re_p)$ valid for a single particle (in which $F(Re_p) = F(Re_0)$ is adopted) are used on its right-hand side. Then the approach leads to relationship

$$Ar\varepsilon^n = F(Re_0) \qquad (26)$$

in which the value of exponent n is determined experimentally. A relation of type (26) can also be obtained by another procedure (Goroshko *et al.* [14], Hartman *et al.* [15]).

Both the approaches have been combined by Xie [16], who chooses the characteristic velocity of system in the same way as our procedure does for the mean velocity u/ε in the voids. Then the relationship of type (26) contains the expression Re_0/ε instead of Re_0. Using the form of dependence $n = F(Re_p)$ determined by Richardson and Zaki [11], Xie [16] determines the course of dependence $Ar\varepsilon^n = F(Re_0/\varepsilon)$ experimentally.

Both the main ways mentioned (in the first one n is dependent on Re whereas in the second n is independent of Re) are also combined in our own approach. Here the value of exponent n in Eq. (23) is expressed as a sum of two terms. The first one is a numerical constant (like in relationships of type (26)), whereas the second depends on the value of the $(1 + \psi_f)$ quantity and hence on the value of Reynolds number (like in relationships of type (25)).

In order to compare the relationships suggested with those taken from literature, we have chosen for the basis the comparison at the porosity value $\varepsilon = 0.68$, which represents the mean value between the minimum and the maximum values ($\varepsilon_{min} \cong 0.42$; $\varepsilon_{max} \cong 0.93$) used for experimental delimitation of validity of the relationships suggested for creeping region of flow [2]. First, using Eq. (14) or (15), we calculate the value of $Re_{b,f}$ number for the chosen values of quantity $\psi_{Re,f}$ (where the value corresponding to the Reynolds number 580 delimiting the laminar region of flow was taken for the maximum).

Second, using Eq. (23), Eqs (19) and (20) with $M_{\psi,f} = 1$, we calculated the values of Re_0 and Ar numbers. From the values found for the Archimedes number, and using the relationship by Hartman *et al.* [17], we calculated the Re_p value.

Using the relationships given in literature for calculation of fluidized bed, we determined the Re_0 value, which we then compared with that found by our procedure using the relative deviation $\delta_{i,2}$

$$\delta_{i,2} = \left(\frac{Re_{0,lit}}{Re_{0,own}} \right) \times 100\ \% \qquad (27)$$

In the comparison with relationships of type (25) we determined the n value with the help of relationships by Richardson and Zaki [11]

$$\begin{aligned} n &= 4.45\,Re_p^{-0.1} \qquad & 1 < Re_p < 500 \\ n &= 2.39 \qquad & Re_p > 500 \end{aligned} \qquad (28)$$

and with the use of relationship by Garside and Al-Dibouni [18]

$$n = \frac{5.09 + 0.2839 Re_p^{0.877}}{1 + 0.104 Re_p^{0.877}} \qquad 10^{-3} < Re_p < 3\times10^4 \qquad (29)$$

In the comparison with the relationships of type (26) we adopted the equation by Wen and Yu [13] (Eq.(30)), that by Goroshko *et al.* [14] (Eq (31)), and that by Hartman *et al.* [15] (Eq. (32)). In the comparison with the equation by Xie [16] (Eq. (33)), the value of exponent n was calculated from Eqs (28).

$$Ar\varepsilon^{4.7} = 18 Re_o + 2.7 Re_o^{1.687} \qquad (30)$$

$$Re_o = \frac{Ar\varepsilon^{4.75}}{18 + 0.6\sqrt{Ar\varepsilon^{4.75}}} \qquad (31)$$

$$Ar\varepsilon^{4.73} = 20.36 Re_o + 1.44 Re_o^{1.805} \qquad (32)$$

$$Ar\varepsilon^{n} = \frac{Re_o}{\varepsilon}\left[18 + \frac{15}{4}\left(\frac{Re_o}{\varepsilon}\right)^{0.75}\right] \qquad (33)$$

The $\psi_{Re,f}$ values chosen and the corresponding values of δ_i deviations are given in Table II. With the same chosen values of the quantity $\psi_{Re,f}$ we also carried out comparisons for the porosity values $\varepsilon = 0.680 \pm 0.22$, which are near the maximum and the minimum porosity values ($\varepsilon_{max} \cong 0.93$; $\varepsilon_{min} \cong 0.42$). From the calculated values of relative deviations $\delta_{i,2}$ and using Eq. (17), we determined the values of mean deviation δ, which are given in Table III for all the porosity values chosen. In this table, the value given for porosity $\varepsilon = 0.46$ and Eq. (25) (where n was calculated using Eq. (29)) is presented along with a value of mean deviation not including the results obtained for the two highest values of $\psi_{Re,f}$ quantity. The values calculated here for Reynolds number Re_p were higher than the value delimiting the validity of Eq. (29). From the table it is obvious that the worst coincidence is that with the oldest relationship (Richardson and Zaki [11]), whereas the best is with the newest one (Xie [16]). At the present, the comparison with the relationships by authors [12,19–21] led to similar results (the mean relative deviations lie in the range from 20 – 25%).

The deviations given in Tables II and III can be compared with those of the $1 + \psi_f$ quantity given in Table I with the help of Eq. (19) with $M_{w,f} = 1$, where n is given by Eq. (23). Thus we can obtain Eq. (34) for the values of deviation in Reynolds number

Table II Comparison of suggested equation with equations given in literature for porosity $\varepsilon = 0.680$ using relative deviation $\delta_{i,2}$

$\psi_{Re,f}$	Eqs, $\delta_{i,2}$					
	(25), (28)	(30)	(31)	(25), (29)	(32)	(33)
0.2	7.0	-1.0	-2.2	-14.5	-5.6	-11.5
0.5	16.4	6.2	8.6	-1.9	5.9	-3.3
0.7	19.6	8.1	13.4	3.1	10.2	-2.3
1	23.9	10.2	19.0	15.0	14.3	-1.8
2	34.6	14.6	29.4	21.2	19.3	-1.8
5	48.0	20.1	30.8	25.4	19.4	-1.0
7	43.4	22.3	26.6	23.1	17.8	-3.0
10	37.4	25.2	20.8	18.9	16.2	-4.2
20	20.4	30.9	6.9	5.0	12.4	-5.5
32	6.2	35.4	-2.5	-7.1	10.1	-5.7

Table III Comparison of suggested equation with equations given in literature using mean relative deviation δ

ε	Eqs, δ, %					
	(25), (28)	(30)	(31)	(25), (29)	(32)	(33)
0.46	37.5	8.9	11.2	18.4	4.6	7.8
0.68	25.7	17.4	16.0	13.5	13.2	4.0
0.90	22.5	21.7	26.2	17.7	20.9	4.5
All data:						
δ, %	28.6	16.0	17.8	16.4	12.9	5.4

$$\delta_{i,2} = \left[\frac{\varepsilon^{2.18 \log(1+0.01\delta_{i,1})}}{1+0.01\delta_{i,1}} - 1 \right] \times 100\ \% \tag{34}$$

For instance, for the mean deviation $\delta \equiv \delta_{i,1} = 3.5\,\%$ and the porosity $\varepsilon = 0.68$ the relative deviation $\delta_{i,2}$ is - 4.6%, for $\varepsilon = 0.9$ it is $\delta_{i,2} = -3.7\,\%$; and for $\varepsilon = 0.46$ it is $\delta_{i,2} = -5.7\,\%$. Comparison of these values with those presented in Tables II and III shows that the deviations from the relationship by Xie [16] *at al.* the porosity values practically coincide with our own experimental error. This is caused by the fact that both the approaches involve one more dimensionless variable, along with relationship for its calculation, as compared with the other approaches.

Moreover, in our relationships there is (additional) resistance number ψ. With regard to the fact that this quantity also appears in the relationship for calculation of Reynolds number $Re_{b,f}$ (Eq. (7)), it is always necessary to use the method of successive approximation in calculations of fluidized bed. On the other hand, it is not necessary to adopt the relationships valid for a single particle. In contrast to the procedures used so far, the approach is devised in a way also allowing application of the relationships suggested to the calculation of flow of GNF through fluidized bed of spherical particles. Like in the case of fixed bed of particles, it is presumed (by adopting the theoretical part of Ref. [1]) that the validity region of the relationships suggested can be extended also to fluidized bed of nonspherical particles.

Conclusion

The equation previously derived for the calculation of incompressible Newtonian fluid-fluidized bed system can also be used in the region of manifestation of the Reynolds number. Among the relationships given in the literature, that by Xie [16] best coincides with the relationships suggested.

Symbols

a numerical coefficient, Eq. (11)
a' numerical coefficient, Eq. (12)
Ar Archimedes number, Eq. (21)
b numerical coefficient, Eq. (11)
C_D drag coefficient, Eq. (8)
d diameter of spherical particle, m

D	column diameter, m
$\dot{D}$	shear rate, s^{-1}
$\dot{D}_{w,f}$	consistency variable for flow through fluidized bed, Eq. (3), s^{-1}
g	gravitational acceleration, $m\,s^{-2}$
l_{ch}	characteristic linear dimension, m
$M_{w,f}$	correction factor for wall effect, Eq. (5)
n	exponent at porosity
N	number of experiments
r_h	hydraulic radius, m
Re_b	Reynolds number for beds of particles, Eq. (6)
Re_o	Reynolds number for fluidized bed, Eq. (22)
Re_p	Reynolds number for a single particle, Eq. (9)
u	superficial velocity, $m\,s^{-1}$
$u_{ch,b}$	($= u/\varepsilon$) characteristic velocity for flow through fixed and fluidized bed, $m\,s^{-1}$
u_p	fall velocity, $m\,s^{-1}$

δ	mean relative deviation
δ_i	relative deviation
ε	porosity of bed
μ	dynamic viscosity, Pa s
μ_e	effective viscosity, Pa s
ρ	density of fluid, $kg\,m^{-3}$
ρ_s	density of particle, $kg\,m^{-3}$
τ	shear stress, Pa
$\tau_{w,f}$	consistency variable for flow through fluidized bed, Eq. (4), Pa
ψ	resistance number
Ω	dimensionless parameter, Eq. (1)

Indexes

b	bed
calc	calculated
crit	critical
exp	experimental
f	fluidized bed
lit	referenced to relationship given in literature
max	maximal
min	minimal
own	referenced to our own relationship

p particle
Re referenced to region of manifestation of Reynolds number

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