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**MODELLING THE CAKE FILTRATION  
ON A CYLINDRICAL ROTARY NUTSCHE**

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*The cylindrical rotary Nutsche is constructed like a circular drum with the bottom half being used as filter area. The drum is placed on four wheels and simple rope drive enables the drum to oscillate. This arrangement should have several advantages in comparison with a classical planar Nutsche. However, for the prediction of filtering performance of the rotary Nutsche, equations of filtration should be used taking into account the variability of filter face area during the filtration period. In this paper, a numerical model has been proposed for the calculation of a constant pressure cake filtration on a cylindrical Nutsche in non-oscillating regime. The results of numerical solution of the model for filtration of diatomite and cellulose suspensions are compared with those obtained in testing the filtering ability of a pilot plant cylindrical rotary Nutsche.*

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## Introduction

For industrial separations of more coarse suspensions, the vacuum or pressure Nutsches are often used [1]. They are principally open or closed vessels with a planar filter medium at the bottom. As a variant of such a classical Nutsche, Pierson developed a cylindrical rotary Nutsche (RN filter) [2]. The rotary Nutsche is constructed like a circular drum with the bottom half being used as filter area. The drum is placed on four wheels and simple rope drive enables the drum to oscillate. It is declared that this arrangement should possess several merits in comparison with the classical Nutsche.

Thick filter cakes created on the planar filter medium in a classical Nutsche have a tendency to crack and hence can be poorly washed and dewatered. A semi-tubular filter bed, which is allowed to swing forwards and backwards, makes it possible for the cake to be formed uniformly and to fill in the cracks as long as the cake maintains a certain degree of fluidity. After a cake has been formed, washing can take place. The drum can be made to rock more vigorously (or inverted) causing the cake to reslurry. If hot air (or gas) is introduced, the RN filter can act like a tumble dryer. For the cake discharge the drum is inverted and an air blast breaks the cake. The entire filter is totally enclosed, satisfying the most stringent safety regulations. A drawback of the RN filter is that during the batch filtration period the filter area decreases and, therefore, the filter performance of RN filter is lower as compared with that of a planar Nutsche. At the same time, the filtering performance of a planar Nutsche can be estimated using the filtration equation, which was developed for a constant pressure filtration with the constant filter face area (e.g. [1]). For the prediction of filtering performance of the RN filter, modified equations of filtration should be used taking into account the variability of filter face area during the filtration period.

In this paper, such equations along with their numerical solution for the filtration of suspensions of diatomite and cellulose are presented for filtration course without drum oscillations. The results of numerical solution are compared with those obtained in testing the filtering ability of a pilot plant cylindrical rotary Nutsche [3].

## Cylindrical Nutsche Performance Modelling

### Governing Equations of Filtration

A batch cake filtration at constant pressure difference on a cylindrical filter cloth attached to the inner wall of drum is considered. Two regimes of filtration are possible: without drum oscillations and with drum oscillations. Herein, the attention is paid to filtration without drum oscillations, which can serve as a first

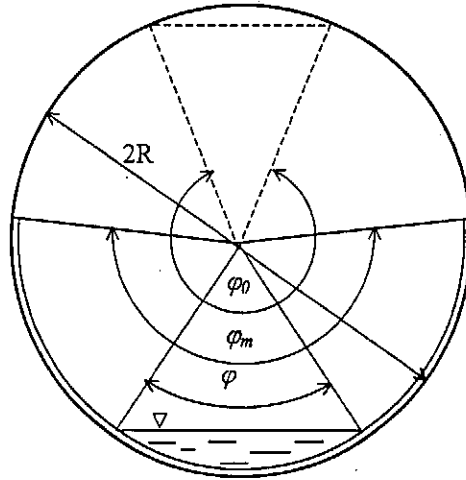


Fig. 1 Schematic diagram of the filter drum geometry

approximation to the rotary Nutsche performance.

The geometry of the filter cloth is shown in Fig. 1. The filtration period is divided into two phases. The first one runs till the whole filter medium is immersed in suspension ( $\varphi \geq \varphi_m$ ). After that the second phase follows.

#### First Phase of Filtration

It is assumed that the thickness  $h$  of filter cake is neglected as compared with the drum radius  $R$  ( $h \ll R$ ) and the filter face area given generally as

$$A = (R - h)L\varphi \quad (1)$$

has the constant value

$$A_0 = RL\varphi_m \quad (2)$$

during this phase. Under this condition, the volume  $V_{s0}$  of suspension in the drum at the beginning of filtration, the volume  $V_s$  of suspension at variable central angle  $\varphi$ , the volume  $V_f$  of filtrate, the volume  $V_{fc}$  of filter cake, and the height  $h$  of filter cake can be expressed as

$$V_{s0} = \frac{R^2 L}{2} (\varphi_0 - \sin \varphi_0) \quad (3)$$

$$V_s = \frac{R^2 L}{2} (\varphi - \sin \varphi) \quad (4)$$

$$V_f = V_{s0} - V_s \quad (5)$$

$$V_{fc} = \frac{V_f x}{(1 - \varepsilon) \rho_s} \quad (6)$$

$$h = \frac{V_{fc}}{A_0} \quad (7)$$

The volume flow rate of filtrate is given by the filtration equation

$$\frac{dV_f}{d\tau} = \frac{CA_0^2}{2(V_f + A_0 V_0)} \quad (8)$$

where  $C$  and  $V_0$  are filtration constants related to the unit filtration area.

Then, the dependences  $V_f = V_f(\tau)$ ,  $V_s = V_s(\tau)$ ,  $\varphi = \varphi(\tau)$ ,  $V_{fc} = V_{fc}(\tau)$  and  $h = h(\tau)$ , which characterize the filtration period course, can be determined by solving the system of differential equations comprising Eq. (8) and the equations

$$\frac{dV_s}{d\tau} = -\frac{dV_f}{d\tau} \quad (9)$$

$$\frac{d\varphi}{d\tau} = \frac{dV_s}{d\tau} \frac{2}{R^2L(1 - \cos\varphi)} \quad (10)$$

$$\frac{dV_k}{d\tau} = \frac{dV_f}{d\tau} \frac{x}{(1 - \varepsilon)\rho_s} \quad (11)$$

$$\frac{dh}{d\tau} = \frac{1}{A_0} \frac{dV_{fc}}{d\tau} \quad (12)$$

obtained by differentiation of Eqs (4) – (7). The corresponding initial conditions are given as

$$\text{for } \tau = 0: \quad V_f = 0, V_s = V_{s0}, \varphi = \varphi_0, V_{fc} = 0, h = 0, A = A_0 \quad (13)$$

### Second Phase of Filtration

In this phase, the reduction of filter face area, which is caused by decreasing level of suspension in the drum and increasing filter cake height, is taken into account. Then the system of differential equations needed for solution has the following form

$$\frac{dV_f}{d\tau} = \frac{CA^2}{2(V_f + AV_0)} \quad (14)$$

$$\frac{dV_s}{d\tau} = -\frac{dV_f}{d\tau} \quad (15)$$

$$\frac{d\varphi}{d\tau} = \frac{dV_s}{d\tau} \left[ 1 - \frac{Lx(R-h)(\varphi - \sin\varphi)}{(1-\varepsilon)\rho_s A} \right] \left[ \frac{2}{(R-h)^2 L (1 - \cos\varphi)} \right] \quad (16)$$

$$\frac{dV_{fc}}{d\tau} = \frac{dV_f}{d\tau} \frac{x}{(1-\varepsilon)\rho_s} \quad (17)$$

$$\frac{dh}{d\tau} = \frac{dV_{fc}}{d\tau} \frac{1}{A} \quad (18)$$

$$\frac{dA}{d\tau} = -\frac{dh}{d\tau} L \varphi + (R-h)L \frac{d\varphi}{d\tau} \quad (19)$$

The corresponding initial conditions are

$$\text{for } \tau = \tau_1: \quad V_f = V_{f1}, \quad V_s = V_{s1}, \quad \varphi = \varphi_1, \quad V_{fc} = V_{fc1}, \quad h = h_1, \quad A = A_0 \quad (20)$$

### Solution Procedure

The Runge–Kutta method of 4<sup>th</sup> order has been used for the solution of the both systems of differential equations (8) – (12) and (14) – (19) with initial conditions (13) and (20) for filtration of suspensions of diatomite and cellulose. The used time steps  $\Delta\tau$  were 0.1 s and 0.01 s. The values of filtration parameters, corresponding to the conditions of testing the filtration ability of the pilot plant cylindrical rotary Nutsche [3], were

- diatomite filtration:  $R = 0.42$  m,  $L = 1.26$  m,  $\varphi_m = 3.15$  rad  $\approx 180^\circ$ ,  
 $A_0 = 0.83$  m<sup>2</sup>,  $V_{s0} = 170$  l, ( $\varphi_0 = 5.24$  rad  $\approx 300^\circ$ ),  $x = 50$  kg m<sup>-3</sup>,  
 $\varepsilon = 0.80$ ,  $V_0 = 0.28$  m, and  $C = 4.95 \times 10^{-4}$  m<sup>2</sup> s<sup>-1</sup>.
- cellulose filtration:  $R = 0.42$  m,  $L = 1.26$  m,  $\varphi_m = 3.15$  rad  $\approx 180^\circ$ ,

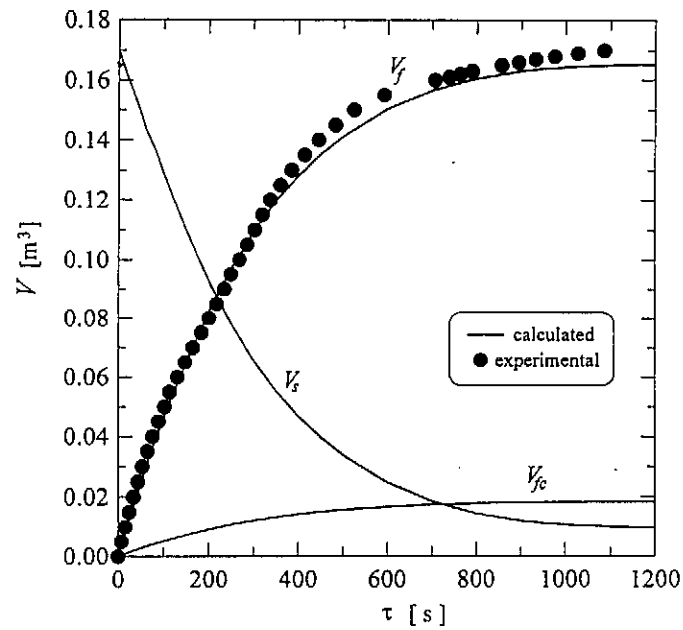
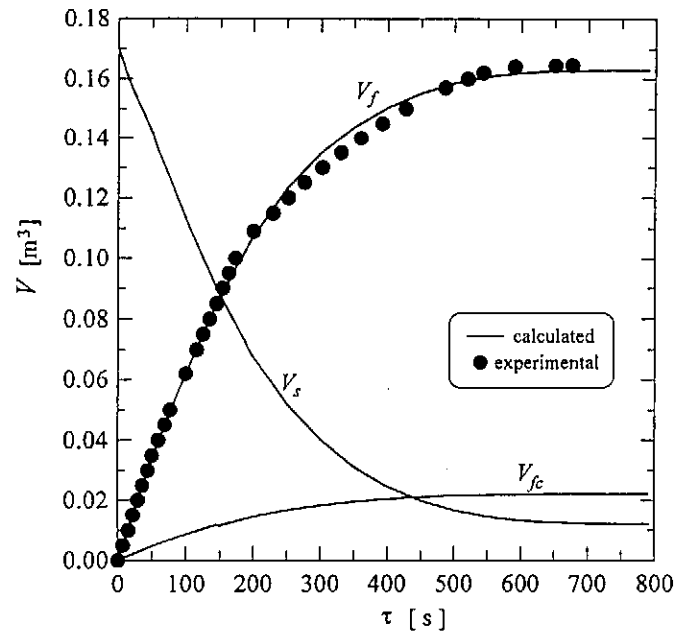


Fig. 2 Dependences of the filtrate volume  $V_f$ , suspension volume  $V_s$ , and filter cake volume  $V_{fc}$  on the time  $\tau$  of filtration: a – diatomite suspension; b – cellulose suspension

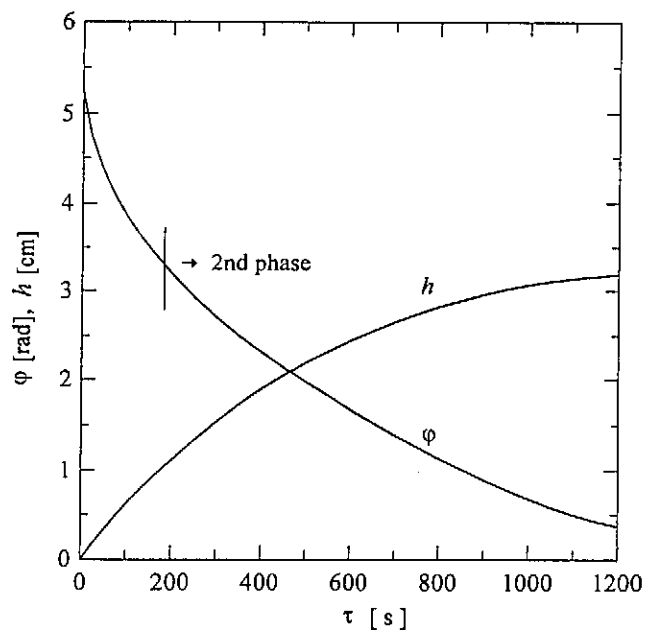
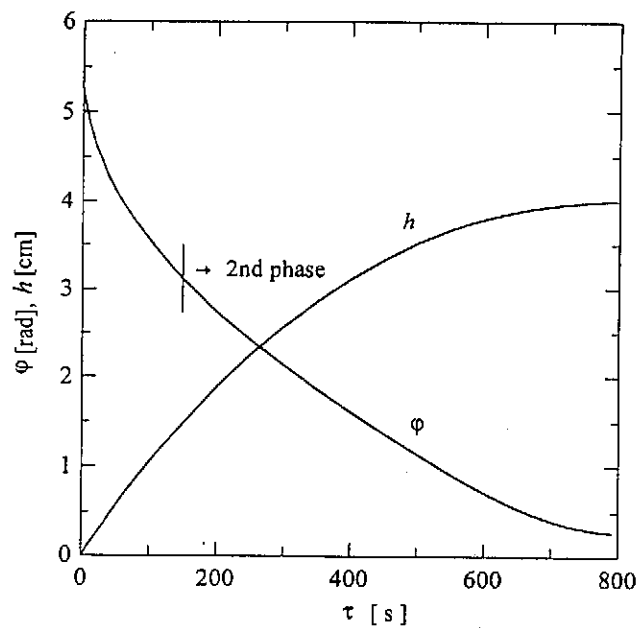


Fig. 3 Dependences of the central angle  $\varphi$  of suspension level and the filter cake height  $h$  on the time  $\tau$  of filtration: a – diatomite suspension; b – cellulose suspension



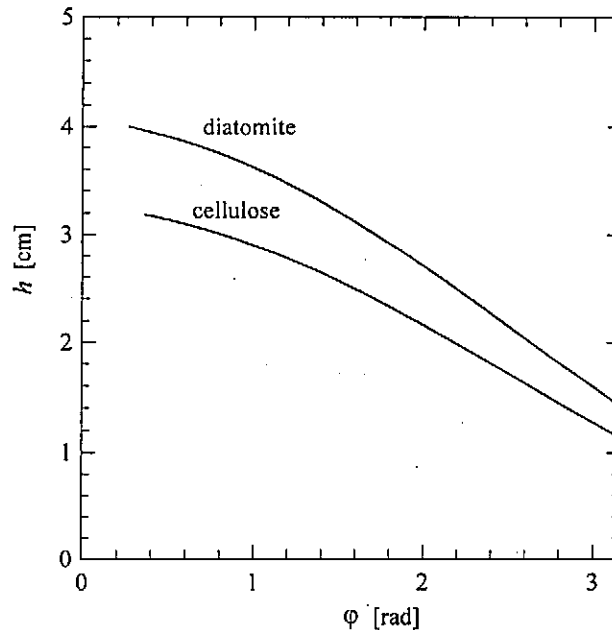


Fig. 4 Dependence of the filter cake height  $h$  on the central angle  $\varphi$  at the end of filtration: ( $\varphi = 0$  rad at the centre of filter cloth,  $\varphi = \varphi_m = 3.15$  rad at the edge of filter cloth, see Fig. 1)

$$A_0 = 0.83 \text{ m}^2, V_{s0} = 170 \text{ l}, (\varphi_0 = 5.24 \text{ rad} \approx 300^\circ), x = 25 \text{ kg m}^{-3}, \\ \varepsilon = 0.85, V_0 = 0.19 \text{ m}, \text{ and } C = 2.5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}.$$

It was found out that the time interval 0.1 s sufficient for the solution of filtration at given conditions.

## Results

The resulting time dependences  $V_f = V_f(\tau)$ ,  $V_s = V_s(\tau)$ , and  $V_{fc} = V_{fc}(\tau)$  are in graphical form given in Fig. 2a for filtration of diatomite suspension and in Fig. 2b for filtration of the cellulose suspension. The dependences  $\varphi = \varphi(\tau)$  and  $h = h(\tau)$ , which indicate the position of the suspension level and the height of filter cake on filtering part of filter cloth, are shown in Figs 3a and 3b. The corresponding dependences  $h = h(\varphi)$  describing the distribution of the height  $h$  of filter cake retained on the filter cloth at the end of filtration are shown in Fig. 4.

In Fig. 2, the calculated dependences  $V_f = V_f(\tau)$  are also compared with those resulting from testing a pilot cylindrical rotary Nutsche [3]. It is evident that the agreement between calculated and experimental data of  $V_f$  is very good and

the proposed model works well for prediction of the filter performance. Figure 4 documents that, in accordance with expectation, the height of filter cake increases from the edge to the centre of the filter cloth.

## Conclusion

A numerical model has been proposed for the solution of a constant pressure difference filtration on a cylindrical Nutsche in non-oscillating regime. The results were presented of the model solution for filtration of diatomite and cellulose suspensions. The validity of the model was confirmed by the very good agreement between calculated and experimental dependences of filtrate volume  $V_f$  on the time  $\tau$  of filtration. Next, the filtration in the oscillating regime will be solved.

## Symbols

$A$	filter face area, $\text{m}^2$
$A_0$	filter cloth area, $\text{m}^2$
$C$	constant of filtration related to the unit filtration area, $\text{m}^2 \text{s}^{-1}$
$h$	filter cake height, m
$L$	length of cylindrical filter face area, m
$R$	radius of cylindrical filter face area, m
$V$	volume, $\text{m}^3$
$V_0$	constant of filtration related to the unit filtration area, m
$x$	concentration of suspension, $\text{kg m}^{-3}$
$\varepsilon$	filter cake porosity
$\rho_s$	solid density, $\text{kg m}^{-3}$
$\tau$	filtration time, s
$\varphi$	central angle corresponding to the immediate level of suspension in the drum, rad
$\varphi_m$	central angle corresponding to the area covered with filter cloth, rad
$\varphi_0$	central angle corresponding to the level of suspension at the beginning of filtration, rad

## Subscripts

$f$	related to the filtrate
$fc$	related to the filter cake
$s$	related to the suspension
0	related to the beginning of filtration
1	related to the end of first filtration phase

## References

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- [2] Pierson H.G.W.: US Patent 5130021, (see also patents GB 2234188, DE 4022469, and J 2201098).
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