

# INTERSECTION OF THE CONICAL AND CYLINDRICAL SURFACES OVER FINITE STRUCTURES OF NUMBERS

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*In the 1<sup>st</sup> part the finite structures of numbers are defined and - by means of transformations - also conical and cylindrical surfaces over these structures are introduced. At the principal difference between the theory of surfaces over these structures and over real numbers points the Remark 1.1. In the 2<sup>nd</sup> part there is the construction of the intersection of such two surfaces, at least one of which is quadratic. Remark 2.3 generalizes the validity of this theory for today-constructed computers.*

## 1. Finite structures of numbers

Let  $n$  be a given integer. Let  $G$  be a finite set of exactly  $n$ -digit decimal numbers of the form

$$(1.1) \quad \pm c_1.c_2c_3\mathbf{K}c_n \times (10)^{\pm\kappa},$$

where  $c_i \in \{0,1,\mathbf{K},9\}$  are ciphers,  $c_1 \neq 0$  for the numbers different from zero and  $\kappa \in \{0,1,\mathbf{K},99\}$ .

Let  $R$  be the set of all real numbers including the four constants  $0,1,\pi,e$ , four binary operations  $+, -, \times, \div$ , the relation  $<$  and ten functions  $\sqrt{\quad}, \sin, \cos, \tan, \arcsin, \arccos, \arctan, \exp, \ln, \text{abs}$ . Let  $T: G \rightarrow R$  be an embedding, i.e. mapping which maps an arbitrary number of  $G$  into the same number of  $R$ . Let  $T': R \rightarrow G$  be such a mapping, which maps the number

$$(1.2) \quad \pm c_1.c_2c_3\mathbf{K}c_nc_{n+1}c_{n+2}\mathbf{K} \times (10)^{\pm\kappa},$$

of  $R$  in an infinite decimal expansion into the number (1.1) of  $G$ .

By using the mappings  $T, T'$  we get in  $G$  associated restrictions of the four constants  $0,1,\pi,e$ , four operations  $+, -, \times, \div$ , the relation  $<$  and above mentioned ten functions. To all this restrictions we leave the original notation. The set  $G$  together with the enumerated four constants, four operations, one relation and ten functions we call a *finite structure of  $n$ -digit decimal numbers*. Because the operations in  $G$  do not fulfil neither the associative nor the distributive law, it is needed to build the theory of the curves and surfaces in  $G^3$  differently than in the space  $R^3$ .

Only the constant functions fulfil over  $G$  the well-known definition of continuity of a function over  $R$ . Let us denote  $\Phi = \{\sqrt{\quad}, \sin, \cos, \tan, \arcsin, \arccos, \arctan, \exp, \ln, \text{abs}\}$  and  $\Omega = \{+, -, \times\}$ . Further let us denote by  ${}^1F$  the class of all such functions  $f(t)$  of one variable  $t \in G$ , for which it holds:

$$(1.3) \quad ((f(t) = \text{const}) \vee (f(t) = t) \vee (f \in \Phi)) \Rightarrow f(t) \in {}^1F;$$

$$(1.4) \quad (f_1, f_2 \in {}^1F) \Rightarrow f_1 * f_2 \in {}^1F \quad \text{for } * \in \Omega \quad \text{and if moreover } f_2 \neq 0, \text{ then also } f_1 \div f_2 \in {}^1F ;$$

$$(1.5) \quad (f_1, f_2 \in {}^1F) \Rightarrow f_1(f_2(t)) \in {}^1F \quad \text{for such } t, \text{ for which } f_1(f_2(t)) \text{ is defined.}$$

Let us denote  ${}^2F$  the class of all such functions  $f(r, s)$  of two variables  $r, s \in \mathbf{G}$ , for which it holds:

$$(1.6) \quad ((f_1(r), f_2(s) \in {}^1F) \wedge ((f(r, s) = f_1(r)) \vee (f(r, s) = f_2(s)))) \Rightarrow f(r, s) \in {}^2F ;$$

$$(1.7) \quad (f_1(r, s), f_2(r, s) \in {}^2F) \Rightarrow (f_1(r, s) * f_2(r, s)) \in {}^2F \quad \text{for } * \in \Omega \quad \text{and if moreover } f_2 \neq 0, \text{ then also } f_1 \div f_2 \in {}^2F ;$$

$$(1.8) \quad ((f_1(t) \in {}^1F) \wedge (f_2(r, s) \in {}^2F)) \Rightarrow f_1(f_2(r, s)) \in {}^2F \quad \text{for such } r, s, \text{ for which } f_1(f_2(r, s)) \text{ is defined;}$$

$$(1.9) \quad (f_1(u, v), f_2(r, s), f_3(r, s) \in {}^2F) \Rightarrow f_1(f_2(r, s), f_3(r, s)) \in {}^2F \quad (\text{compare [1] }).$$

By a *curve* in the space  $\mathbf{G}^3$  we understand such a set of all points  $(f_1(t), f_2(t), f_3(t))$  that  $f_i(t) \in {}^1F$  are functions of one variable  $t \in \langle \tau, \vartheta \rangle \subset \mathbf{G}$ ;  $\tau, \vartheta \in \mathbf{G}$ . By a *surface* in the space  $\mathbf{G}^3$  we understand such a set of all ordered triplets  $(f_1(t, s), f_2(t, s), f_3(t, s))$  that  $f_i(t, s) \in {}^2F$  are functions of two variables  $t \in \langle \tau, \vartheta \rangle \subset \mathbf{G}$ ,  $s \in \langle \sigma, \xi \rangle \subset \mathbf{G}$ ;  $\tau, \vartheta, \sigma, \xi \in \mathbf{G}$ . For the curves and surfaces we will use the usual parametric equations

$$(1.10) \quad x = f_1(t), \quad y = f_2(t), \quad z = f_3(t), \quad t \in \langle \tau, \vartheta \rangle \subset \mathbf{G}, \quad \tau, \vartheta \in \mathbf{G},$$

$$(1.11) \quad x = f_1(t, s), \quad y = f_2(t, s), \quad z = f_3(t, s), \quad t \in \langle \tau, \vartheta \rangle \subset \mathbf{G}, \quad s \in \langle \sigma, \xi \rangle \subset \mathbf{G}, \quad \tau, \vartheta, \sigma, \xi \in \mathbf{G} \text{ respectively.}$$

By a *conical surface* in the space  $\mathbf{G}^3$  with a *base curve*  $K: (f_1(t), f_2(t), f_3(t))$ ,  $t \in \langle \tau, \vartheta \rangle$  and with a *vertex*  $V = (v_1, v_2, v_3)$  we understand a surface with parametric equations

$$(1.12) \quad \begin{aligned} x &= f_1(t) + s(v_1 - f_1(t)) \\ y &= f_2(t) + s(v_2 - f_2(t)), \quad t \in \langle \tau, \vartheta \rangle, \quad s \in \langle \sigma, \xi \rangle. \\ z &= f_3(t) + s(v_3 - f_3(t)) \end{aligned}$$

By a *cylindrical surface* in the space  $\mathbf{G}^3$  with a *base curve*  $K: (f_1(t), f_2(t), f_3(t))$ ,  $t \in \langle \tau, \vartheta \rangle$  and with a *direction vector*  $\vec{v} = (v_1, v_2, v_3)$  we understand a surface with parametric equations

$$(1.13) \quad \begin{aligned} x &= f_1(t) + sv_1 \\ y &= f_2(t) + sv_2, \quad t \in \langle \tau, \vartheta \rangle, \quad s \in \langle \sigma, \xi \rangle. \\ z &= f_3(t) + sv_3 \end{aligned}$$

By a *quadratic conical* or *quadratic cylindrical surface* we understand such a *conical* or *cylindrical surface*, the *base curve* of which is - in the plane  $z=0$  - one of the further mentioned *conic sections* ( $a, b, r$  are positive constants):

$$(1.14) \quad \text{circle} \quad x = r \cdot \cos t, \quad y = r \cdot \sin t, \quad t \in \langle 0, 2\pi \rangle ;$$

$$(1.15) \quad \text{ellipse} \quad x = a \cdot \cos t, \quad y = b \cdot \sin t, \quad t \in \langle 0, 2\pi \rangle ;$$

(1.16) *hyperbola*  $x = a/\cos t$ ,  $y = b \cdot \tan t$ ,  $t \in (-\pi/2, \pi/2)$  or  $t \in (\pi/2, 3\pi/2)$ ;

(1.17) *parabola*  $x = t$ ,  $y = at^2$ ,  $t \in \langle \tau, \vartheta \rangle$ ,

where the 3<sup>rd</sup> coordinate  $v_3$  of the vertex  $V = (v_1, v_2, v_3)$  or of the direction vector  $\vec{p} = (v_1, v_2, v_3)$  is different from zero.

**Remark 1.1.** The curves and surfaces over  $G$  depends on the position with respect to the system of coordinates; in great distances from the origin the deformations are serious.

Let us denote  $x_1 = 1\,234\,567\,890$ ;  $y_1 = 5\,432\,109\,876$ ;  $z_1 = 7\,890\,123\,456$ .

**Example 1.1.** If  $G$  has  $n=10$  ciphers, then  $2\pi = 6.283185307$ ; the circle  $x = \cos t$ ,  $y = \sin t$ ,  $t \in \langle 0, 2\pi \rangle$  consists of 6 283 185 307 points, but the translated circle  $x = x_1 + \cos t$ ,  $y = y_1 + \sin t$ ,  $t \in \langle 0, 2\pi \rangle$  consists of 5 points only (see Fig.1).

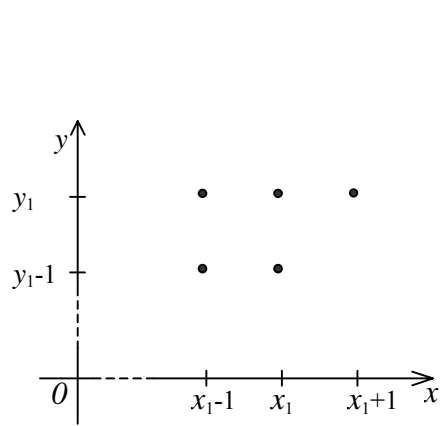


Fig. 1

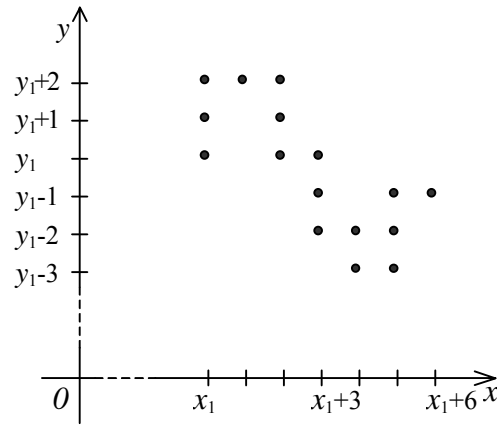


Fig. 2

**Example 1.2.** If  $G$  has  $n=10$  ciphers, then the 1<sup>st</sup> turn of the helix  $x = \cos t$ ,  $y = \sin t$ ,  $z = t$ ,  $t \in \langle 0, 2\pi \rangle$  consists of 6 283 185 307 points, but the translated curve  $x = x_1 + \cos t$ ,  $y = y_1 + \sin t$ ,  $z = z_1 + t$ ,  $t \in \langle 0, 2\pi \rangle$  has only the following 11 points:  $[x_1 + 1, y_1, z_1]$ ;  $[x_1, y_1, z_1]$ ;  $[x_1, y_1, z_1 + 1]$ ;  $[x_1 - 1, y_1, z_1 + 1]$ ;  $[x_1 - 1, y_1, z_1 + 2]$ ;  $[x_1 - 1, y_1, z_1 + 3]$ ;  $[x_1 - 1, y_1 - 1, z_1 + 3]$ ;  $[x_1 - 1, y_1 - 1, z_1 + 4]$ ;  $[x_1, y_1 - 1, z_1 + 4]$ ;  $[x_1, y_1 - 1, z_1 + 5]$ ;  $[x_1, y_1 - 1, z_1 + 6]$ .

**Example 1.3.** If  $G$  has  $n=8$  ciphers, then  $2\pi = 6,2831853$ , the curve  $x = t$ ,  $y = 2,5 \cdot \sin t$ ,  $t \in \langle 0, 2\pi \rangle$  has 62831853 points, but the translated curve  $x = x_1 + t$ ,  $y = y_1 + 2,5 \cdot \sin t$ ,  $t \in \langle 0, 2\pi \rangle$  has exactly 16 points (see Fig.2). ( We get these sixteen points successively for  $t = 0$ ,  $t \in (0; \arcsin 0,4)$ ,  $t \in (\arcsin 0,4; \arcsin 0,8)$ ,  $t \in (\arcsin 0,8; 1)$ ,  $t \in (1; 2)$ ,  $t \in (2; \pi - \arcsin 0,8)$ ,  $t \in (\pi - \arcsin 0,8; \pi - \arcsin 0,4)$ ,  $t \in (\pi - \arcsin 0,4; 3)$ ,  $t \in (3; \pi)$ ,  $t \in (\pi; \pi + \arcsin 0,4)$ ,  $t \in (\pi + \arcsin 0,4; 4)$ ,  $t \in (4; \pi + \arcsin 0,8)$ ,  $t \in (\pi + \arcsin 0,8; 5)$ ,  $t \in (5; 2\pi - \arcsin 0,8)$ ,  $t \in (2\pi - \arcsin 0,8; 2\pi - \arcsin 0,4)$ ,  $t \in (2\pi - \arcsin 0,4; 6)$ ,  $t \in (6; 2\pi)$ .)

**Example 1.4.** If  $G$  has  $n=10$  ciphers, then  $2\pi = 6,283185307$ , and the ellipse  $x = 0,82 \cdot \cos t$ ,  $y = 0,37 \cdot \sin t$ ,  $t \in \langle 0, 2\pi \rangle$  has 6 283 185 307 points, but the translated ellipse

$x = 1234567890 + 0,82 \cdot \cos t$ ,  $y = 5432109876 + 0,37 \cdot \sin t$ ,  $t \in \langle 0, 2\pi \rangle$  consists of 4 points only.

## 2. Intersection of two conical and cylindrical surfaces, one of which is quadratic.

**Example 2.1.** Let there be given a quadratic conical surface  $P$  by a base circle  $K$  with parametric equations (1.14) and a vertex  $V = (v_1, v_2, v_3)$ ,  $v_3 \neq 0$ . Let there be given a conical surface  $P_1$  by a curve  $K_1$  with parametric equations

$$(2.1) \quad x = g_1(u), \quad y = g_2(u), \quad z = g_3(u); \quad u \in \langle \eta, \xi \rangle,$$

and with a vertex  $W = (w_1, w_2, w_3)$ . By using the mappings  $T, T'$  we pass from the equations over  $\mathbf{G}$  to formally coincident equations over  $\mathbf{R}$ . Let us put  $A = (x, y, z)$ ,  $A^* = (x^*, y^*, z^*)$ . According to (1.4) we have

$$(2.2) \quad A \in K \Leftrightarrow x^2 + y^2 - r^2 = 0 \quad \wedge \quad z = 0.$$

If  $A \in K$ , then (1.12) implies

$$A^* \in P \Leftrightarrow \begin{cases} x^* = x + s(v_1 - x) \\ y^* = y + s(v_2 - y) \\ z^* = sv_3 \end{cases}$$

and from there

$$s = \frac{z^*}{v_3}, \quad x = \frac{x^* - sv_1}{1-s}; \quad y = \frac{y^* - sv_2}{1-s} \quad \text{for } z^* \neq v_3. \quad \text{By substitution into (2.2) we get}$$

$$(2.3) \quad A^* \in P \Leftrightarrow (x^* - sv_1)^2 + (y^* - sv_2)^2 = r^2(1-s)^2,$$

where  $v_3 \neq 0$ ,  $z^* \neq v_3$ .

If  $A \in K_1$ , then by (2.1) and (1.12) we get  $A^* \in P_1$  if and only if

$$(2.4) \quad \begin{cases} x = g_1(u); & y = g_2(u); & z = g_3(u); & u \in \langle \eta, \xi \rangle \\ x^* = x + m(w_1 - x); & y^* = y + m(w_2 - y); & z^* = z + m(w_3 - z); \end{cases}$$

If we substitute from (2.4) into (2.3), we get for  $A \in P \cap P_1$  the equation

$$\left[ x + m(w_1 - x) - (z + m(w_3 - z))v_1/v_3 \right]^2 + \left[ y + m(w_2 - y) - (z + m(w_3 - z))v_2/v_3 \right]^2 - r^2 \left[ 1 - (z + m(w_3 - z))/v_3 \right]^2 = 0, \quad \text{and from this}$$

$$(2.5) \quad (mc_1 + c_2)^2 + (mc_3 + c_4)^2 - r^2(mc_5 + c_6) = 0,$$

where  $c_1 = w_1 - x - w_3v_1/v_3 + zv_1/v_3$ ;  $c_2 = x - zv_1/v_3$ ;  $c_3 = w_2 - y - w_3v_2/v_3 + zv_2/v_3$ ;  $c_4 = y - zv_2/v_3$ ;  $c_5 = (z - w_3)/v_3$ ;  $c_6 = 1 - z/v_3$ . From (2.5) we get a quadratic equation

$$(2.6) \quad c_7m^2 + c_8m + c_9 = 0,$$

where  $c_7 = c_1^2 + c_3^2 - r^2c_5^2$ ;  $c_8 = 2c_1c_2 + 2c_3c_4 - 2r^2c_5c_6$ ;  $c_9 = c_2^2 + c_4^2 - r^2c_6^2$  and  $c_{10} = c_8^2 - 4c_7c_9$  is the discriminant of the equation (2.6).

For  $c_{10} \geq 0$  and  $c_7 \neq 0$  we get the solutions of quadratic equation (2.6) in the form  $m = \frac{-c_8 \pm \sqrt{c_{10}}}{2c_7}$ . If these are substituted instead of  $m$  into (2.4), we get the asked results for  $x^*$ ,  $y^*$ ,  $z^*$ . By using the mappings  $T, T'$  we pass from the equations over  $\mathbf{R}$  to formally coincident equations over  $\mathbf{G}$  for the asked intersection  $P \cap P_1$ :

$$\begin{aligned}
(2.7) \quad & r > 0; \quad V = (v_1, v_2, v_3), \quad v_3 \neq 0; \quad W = (w_1, w_2, w_3); \quad x = g_1(u), \quad y = g_2(u), \quad z = g_3(u), \quad u \in \langle \eta, \xi \rangle; \\
& c_1 = w_1 - x - w_3 v_1 / v_3 + z v_1 / v_3; \quad c_2 = x - z v_1 / v_3; \quad c_3 = w_2 - y - w_3 v_2 / v_3 + z v_2 / v_3; \\
& c_4 = y - z v_2 / v_3; \quad c_5 = (z - w_3) / v_3; \quad c_6 = 1 - z / v_3; \quad c_7 = c_1^2 + c_3^2 - r^2 c_5^2; \\
& c_8 = 2c_1 c_2 + 2c_3 c_4 - 2r^2 c_5 c_6; \quad c_9 = c_2^2 + c_4^2 - r^2 c_6^2; \quad c_{10} = c_8^2 - 4c_7 c_9; \\
& m = \frac{-c_8 \pm \sqrt{c_{10}}}{2c_7} \quad \text{for } c_{10} \geq 0 \text{ and } c_7 \neq 0;
\end{aligned}$$

$$x^* = x + m(w_1 - x); \quad y^* = y + m(w_2 - y); \quad z^* = z + m(w_3 - z).$$

**Example 2.2.** Let there be given a quadratic conical surface  $P$  by a base circle  $K$  with parametric equations (1.14) and a vertex  $V = (v_1, v_2, v_3)$ ,  $v_3 \neq 0$ . Let there be given a cylindrical surface  $P_1$  by a curve  $K_1$  with parametric equations (2.1) and with a direction vector  $\vec{w} = (w_1, w_2, w_3)$ . For the intersection  $P \cap P_1$  we get similarly as in the example 2.1 the following:

$$\begin{aligned}
(2.8) \quad & r > 0; \quad V = (v_1, v_2, v_3), \quad v_3 \neq 0; \quad \vec{w} = (w_1, w_2, w_3); \quad x = g_1(u), \quad y = g_2(u), \quad z = g_3(u), \quad u \in \langle \eta, \xi \rangle; \\
& c_1 = w_1 - w_3 v_1 / v_3; \quad c_2 = x - z v_1 / v_3; \quad c_3 = w_2 - w_3 v_2 / v_3; \\
& c_4 = y - z v_2 / v_3; \quad c_5 = -w_3 / v_3; \quad c_6 = 1 - z / v_3 \quad \text{and} \\
& c_7, c_8, c_9, c_{10}, m, x^*, y^*, z^* \text{ are the same as in (2.7)}.
\end{aligned}$$

**Example 2.3.** Let there be given a quadratic cylindrical surface  $P$  by a base circle  $K$  with parametric equations (1.14) and a direction vector  $\vec{v} = (v_1, v_2, v_3)$ ,  $v_3 \neq 0$ . Let there be given a conical surface  $P_1$  by a curve  $K_1$  with parametric equations (2.1) and with a vertex  $W = (w_1, w_2, w_3)$ . For the intersection  $P \cap P_1$  we get similarly as in the example 2.1 the following:

$$\begin{aligned}
(2.9) \quad & \vec{v} = (v_1, v_2, v_3), \quad v_3 \neq 0; \quad W = (w_1, w_2, w_3); \quad x = g_1(u), \quad y = g_2(u), \quad z = g_3(u), \quad u \in \langle \eta, \xi \rangle; \\
& c_1 = w_1 - x - w_3 v_1 / v_3 + z v_1 / v_3; \quad c_2 = x - z v_1 / v_3; \quad c_3 = w_2 - y - w_3 v_2 / v_3 + z v_2 / v_3; \\
& c_4 = y - z v_2 / v_3; \quad c_5 = 0; \quad c_6 = 1 \quad \text{and} \\
& c_7, c_8, c_9, c_{10}, m, x^*, y^*, z^* \text{ are the same as in (2.7)}.
\end{aligned}$$

**Example 2.4.** Let there be given a quadratic cylindrical surface  $P$  by a base circle  $K$  with parametric equations (1.14) and a direction vector  $\vec{v} = (v_1, v_2, v_3)$ ,  $v_3 \neq 0$ . Let there be given a cylindrical surface  $P_1$  by a curve  $K_1$  with parametric equations (2.1) and with a direction vector  $\vec{w} = (w_1, w_2, w_3)$ . For the intersection  $P \cap P_1$  we get similarly as above:

$$\mathcal{P} = (v_1, v_2, v_3), v_3 \neq 0; \quad \mathcal{W} = (w_1, w_2, w_3); \quad x = g_1(u), \quad y = g_2(u), \quad z = g_3(u), \quad u \in \langle \eta, \xi \rangle;$$

$$c_1 = w_1 - w_3 v_1 / v_3; \quad c_2 = x - z v_1 / v_3; \quad c_3 = w_2 - w_3 v_2 / v_3$$

$$(2.10) \quad c_4 = y - z v_2 / v_3; \quad c_5 = 0; \quad c_6 = 1 \quad \text{and}$$

$$c_7, c_8, c_9, c_{10}, m, x^*, y^*, z^* \text{ are the same as in (2.7).}$$

**Remark 2.1.** The examples for quadratic conical or cylindrical surface  $P$  with a base conic  $K$  with the equations (1.15) or (1.16) or (1.17) instead of (1.14) and for conical or cylindrical surface  $P$  with a curve  $K_1$  with parametric equations (2.1) are solved similarly.

**Remark 2.2.** By using the equations (2.7) - (2.10) it is immediately possible to write a program for calculation of coordinates  $x^*, y^*, z^*$  of points  $A^* \in P \cap P_1$ . It is more effective to map the surfaces  $P, P_1$  and also their intersection  $P \cap P_1$  into the plane by using the transformation equations of the parallel mapping

$$(2.11) \quad \begin{aligned} p &= r_0 x + r_2 y + r_4 z \\ q &= r_1 x + r_3 y + r_5 z \end{aligned}$$

where  $r_i$  are suitably chosen constants,  $x, y, z$  are right-angled coordinates of the point  $A$  in the space and  $p, q$  are right-angled coordinates of its image in the plane. In this way obtained pictures understood over  $\mathbf{R}$  have some inaccuracies, but the same understood over  $\mathbf{G}$  are accurate.

**Remark 2.3.** Most of up-to-date computers is constructed so that the  $n^{\text{th}}$  valid digit is after operation rounded by the  $(n+1)^{\text{st}}$  digit, where the  $(n+2)^{\text{nd}}$  digit is neglected. It is sufficient change suitably the mapping  $T': \mathbf{R} \rightarrow \mathbf{G}$  and we obtain corresponding finite structure  $\mathbf{G}$  of  $n$ -digit decimal numbers. The important claim of Remark 1.1 stays valid – it is sufficient to note the example 1.4 and the next example.

**Example 2.5.** Let us take a computer, which rounds the  $10^{\text{th}}$  digit according to  $11^{\text{th}}$  digit. We have  $n=10$ ,  $2\pi = 6.283185307$ ;  $x_1 = 1234567890$ ;  $y_1 = 5432109876$ ;  $z_1 = 7890123456$ . Similarly as in the Examples 1.1, 1.2 we consider the translated circle  $x = x_1 + \cos t$ ,  $y = y_1 + \sin t$ ,  $t \in \langle 0, 2\pi \rangle$  and the  $1^{\text{st}}$  turn of the translated helix  $x = x_1 + \cos t$ ,  $y = y_1 + \sin t$ ,  $z = z_1 + t$ ,  $t \in \langle 0, 2\pi \rangle$ . We can easy to obtain the following values:

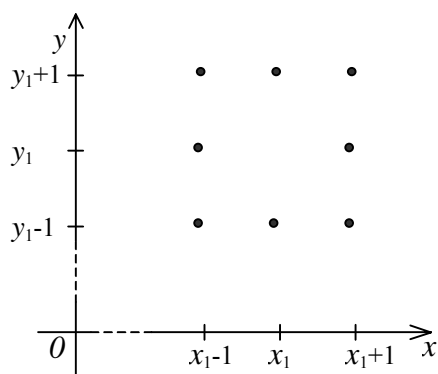


Fig. 3

<i>Interval</i>	$[ x ; y ; z ]$
$0 \leq t \leq 0.499\,999\,999$	$[x_1 + 1 ; y_1 ; z_1]$
$0.5 \leq t \leq 0.523\,598\,774$	$[x_1 + 1 ; y_1 ; z_1 + 1]$
$0.523\,598\,775 \leq t \leq 1.047\,197\,551$	$[x_1 + 1 ; y_1 + 1 ; z_1 + 1]$
$1.047\,197\,552 \leq t \leq 1.499\,999\,999$	$[x_1 ; y_1 + 1 ; z_1 + 1]$
$1.5 \leq t \leq 2.105\,981\,117$	$[x_1 ; y_1 + 1 ; z_1 + 2]$
$2.105\,981\,118 \leq t \leq 2.499\,999\,999$	$[x_1 - 1 ; y_1 + 1 ; z_1 + 2]$
$2.5 \leq t \leq 2.617\,993\,878$	$[x_1 - 1 ; y_1 + 1 ; z_1 + 3]$
$2.617\,993\,879 \leq t \leq 3.499\,999\,999$	$[x_1 - 1 ; y_1 ; z_1 + 3]$
$3.5 \leq t \leq 3.676\,777\,444$	$[x_1 - 1 ; y_1 ; z_1 + 4]$
$3.676\,777\,445 \leq t \leq 4.177\,204\,189$	$[x_1 - 1 ; y_1 - 1 ; z_1 + 4]$
$4.177\,204\,190 \leq t \leq 4.499\,999\,999$	$[x_1 ; y_1 - 1 ; z_1 + 1]$
$4.5 \leq t \leq 5.235\,987\,755$	$[x_1 ; y_1 - 1 ; z_1 + 5]$
$5.235\,987\,756 \leq t \leq 5.499\,999\,999$	$[x_1 + 1 ; y_1 - 1 ; z_1 + 5]$
$5.5 \leq t \leq 5.748\,000\,516$	$[x_1 + 1 ; y_1 - 1 ; z_1 + 6]$
$5.748\,000\,517 \leq t \leq 6.283\,185\,307$	$[x_1 + 1 ; y_1 ; z_1 + 6]$ .

It is easy to see from these values, that the translated circle has exactly 8 points (see Fig.3) and the 1<sup>st</sup> turn of the translated helix has exactly 15 points.

### References

[1] Kateřinák, J.: Teorie nad konečnými strukturami  $G$ -čísel. *Zborník 6.vedeckej konferencie VŠDS v Žiline*, 1979, 97-101, [in Czech].

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