AN EFFECTIVE APPROACH TO ROUTING PROBLEMS

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Abstract
The Clarke -Wright method is one of the commonly applied heuristic methods used for finding solutions of the vehicle routing problems. This paper describes differential modification of the original Clarke-Wright method.

1. INTRODUCTION

Like the well-known traveling salesman problem, the vehicle routing problem is really fascinating. Easy to describe, but difficult to solve. One might say that such a problem belongs to the class of NP-hard problems.

The original and classical vehicle routing problem can be traced back in time many hundreds of years. This problem arises every year at about the same time – at Christmas time. The problem is as follows: How does Santa Claus can do it? Naturally, Santa has a single vehicle with finite capacity that leaves from a single depot. Millions of stochastic demands having tight time windows must be satisfied within a 24-hour period. Our wonderment in how the solution is obtained is stemming from our childhood. Of course that motivates our search for algorithms to solve the vehicle routing problem.

Obviously, the most fascinating feature of the vehicle routing is that the basic problem can be extended into great number of variations in the real-world problems. For instance garbage collection, milk or mail delivery, parking meter collection, electrical lines inspection and gas distribution system inspection, ect.

Let us consider the distribution problem with a single vehicle of capacity W. The vehicle is based at the central depot. The customers are indexed by number i=1, 2, .. , n. In addition i=0 refers to the depot. A customer i has a demand of q_i ($q_i < W$). Let us denote the travel distance between customer i and customer j by d_{ij}. Our task is to route the vehicle (starting and finishing at the depot) so that demands of all customers are satisfied and at the same time, the total travel distance is minimized.

The problem described above belongs to the class of problems that are difficult to solve exactly. There are methods that can find a good solution but not the best solution at all. In Janacek [3], [1] a survey of heuristic algorithms for solving a number of practical routing problems is presented. The most successful and commonly used methods for finding solution to the vehicle routing problem are the ones that are heuristic in nature, the Clarke-Wright method being very popular [2].

2. CLARKE-WRIGHT ALGORITHM FOR TRAVELING SALESMAN PROBLEM

For our purpose we will consider a single vehicle routing problem without capacity restriction. We denote by $\mathbf{D}=(d_{ij})$ the distance matrix, where d_{ij} is the distance between customer i and customer j. There are n customers, whose demands have to be satisfied. The objective is to find a tour, which passes through each customer at least once and returns to the starting point (depot) and which is shorter than every other such a tour. This task is the well-known traveling salesman problem.

To solve this problem we have to find a cyclic permutation π of n+1 elements 0, 1, 2, .., n which minimizes the sum

$$d(\pi) = \sum_{i=0}^{n} d[i, \pi(i)], \qquad (1)$$

where π(i) denotes the number of customers which have to be visited after the customer i. The original Clarke-Wright algorithm initially starts by assuming that a single vehicle serves a single customer. For a set of n customers Clarke-Wright method assumes that n vehicles will be required. The total cost of the initial solution is

$$\sum_{k=1}^{n} (d_{0k} + d_{k0}). \qquad (2)$$

Afterwards, the procedure calculates the savings s_{ij}. More concretely saving in distance that can be obtained by merging tours 0→i→0 and 0→j→0, and serving customers i and j with one single vehicle in the tour 0→i→j→0. When the distance between customer i and customer j is indicated by d_{ij}, the savings s_{ij} are calculated by using the following formula

$$s_{ij} = d_{i0} + d_{oj} - d_{ij} \qquad (3)$$

Note that the savings s_{ij} are defined only if i is the last (or the only one) and j is the first (or the only one) element on obtained tours. The calculated savings s_{ij} are sorted in decreasing order. Thereafter the customer i and j with the highest savings are merged together as long as no more mrging can be done.

3. DIFFERENTIAL MODIFICATION OF CLARKE-WRIGHT METHOD

The original Clarke-Wright method belongs to the sort of greedy methods. Any greedy algorithm works by making the decision that seems to be the most promising at any moment. So the Clarke-Wright algorithm is to merge cycles with high savings. At the end of the algorithm the unprofitable cycles are merged.

Our differential modification of Clarke-Wright algorithm merges feasible routes according to the highest differences between the highest savings in colums and in rows of the matrix **S**=(s_{ij}). Thus the formal algorithm may be presented as follows:

Step 1: Initialize

Set I=J={1, 2, .., n}

$\pi(i) = a_i = b_i = i$ for $i \in I$

$\mathbf{S} = (s_{ij})$ for $i \in I, j \in J$,

where s_{ij} are savings calculated by (3).

Step 2: Merge

a) Let u_i be a column difference and v_j be a row difference. Differences are calculated using the following formulas:

$$u_i = s_{ir} - x_i \quad \text{for } i \in I,$$

where

$$s_{ir} = \max_{j \in J}(s_{ij})$$

$$x_i = \max\left[\max_{j \in J-r}(s_{ij}), s(b_r, a_i)\right].$$

$$v_j = s_{tj} - y_j \qquad \text{for } j \in J,$$

where

$$s_{tj} = \max_{i \in I}(s_{ij})$$

$$y_j = \max\left[\max_{i \in I-t}(s_{ij}), s(b_j, a_t)\right].$$

b) Put $v_{j^*} = \max_{j \in J}(v_j)$

$$u_{i^*} = \max_{i \in I}(u_i)$$

c1) If $u_{i^*} > v_{j^*}$ then put

$$\pi(i^*) = r, \quad s(b_r, a_{i^*}) = -\infty, \quad x = a_{i^*}, \quad y = b_r, \quad a_x = y, \quad b_y = x$$

$$I = I - i^*, \quad J = J - r$$

c2) If $u_{i^*} < v_{j^*}$ then put

$$\pi(t) = j^*, \quad s(b_{j^*}, a_t) = -\infty, \quad x = a_t, \quad y = b_{j^*}, \quad a_x = y, \quad b_y = x,$$

$$I = I - t, \quad J = J - j^*$$

c3) If $u_{i^*} = v_{j^*}$ an $s(b_r, a_{i^*}) < s(b_{j^*}, a_t)$, then go to **Step 2** c1, else go to **Step 2** c2.

The merging described in **Step 2** is repeated (n-1)-times.

Step 3: <u>End</u>

Set $\pi(i) = 0$ for $I = \{i\}$

$\pi(0) = j$ for $J = \{j\}$

and stop.

Since the main diagonal of the matrix **S**=(s_{ij}) is not- zmena slovosledu defined in **Step 1** we can set formally $s_{ii} = -\infty$. An array **a**=(a_i) is created during the first tour from the depot 0 to the customer i.

4. CONCLUSIONS AND FURTHER REMARKS

The vehicle routing problem is an interesting combinatorial problem with many real-world applications. In this paper the differential modification of Clarke-Wright algorithm was introduced. Many computational experiments were done with both the methods. Distance matrixes with elements uniformly distributed on interval <0, 1> were randomly generated. On average the computational tests yielded a 23 percent savings in the length of tours. Nearly always the differential modification of Clarke-Wright algorithm yields better solution than the original method.

There is no problem to use the differential modification for solving a capacitated routing problem with a single vehicle – capacity restrictions are added. Hopefully the presented method can be useful for many approximate methods based on greedy principle. One of such methods is an Augment-Merge algorithm described in [2], [4]. It is relatively widespread algorithm for solving arc capacitated routing problems.

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